LSI vs Link Analysis
(A Survey)

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Raleigh, NC

1/23/2003

Outline

- **Background & History**

Outline

- Background & History
- Vector Space Approach

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- Link Analysis Approach

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- Hybrid Approachs

Background

Goal

- Identify documents that best match users query

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- Recall $= \dfrac{\#relevant\ docs\ retrieved}{\#docs\ in\ collection}$ (max # useful docs)

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Do it *FAST!*

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SMART

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- Based on matrix methods

Term–Document Matrix

Start With Dictionary of Terms

- Single words — or short phrases (e.g., $landing\ gear$)

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- **A** is sparse — but otherwise unstructured
- **A** contains a lot of uncertainty

Query Matching

Query Vector

- $\mathbf{q}^T = (q_1, q_2, \ldots, q_m)$ where $q_i = \begin{cases} 1 & \text{if Term } i \text{ is requested} \\ 0 & \text{if not} \end{cases}$

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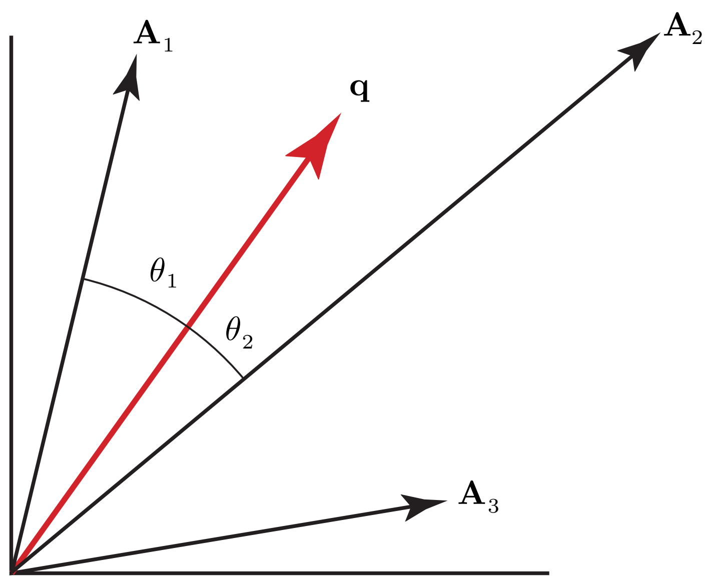

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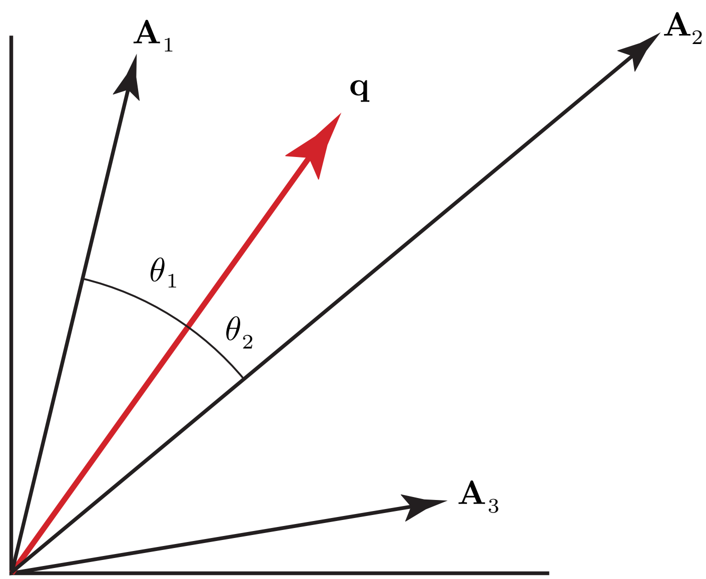

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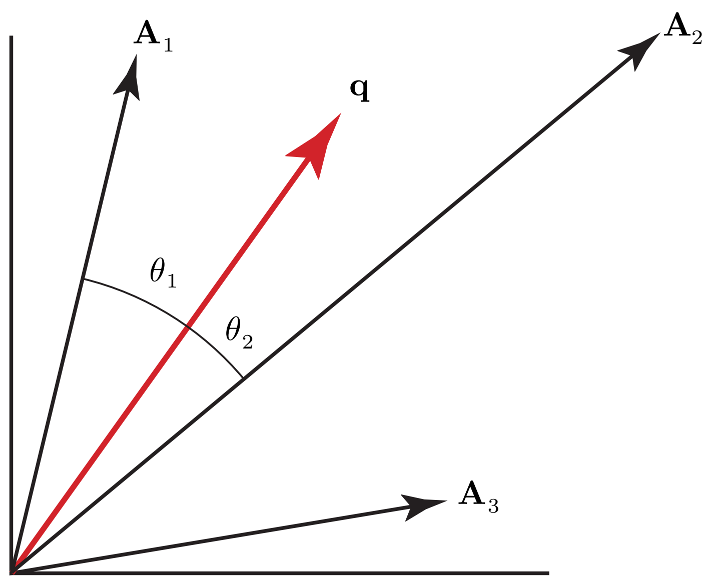

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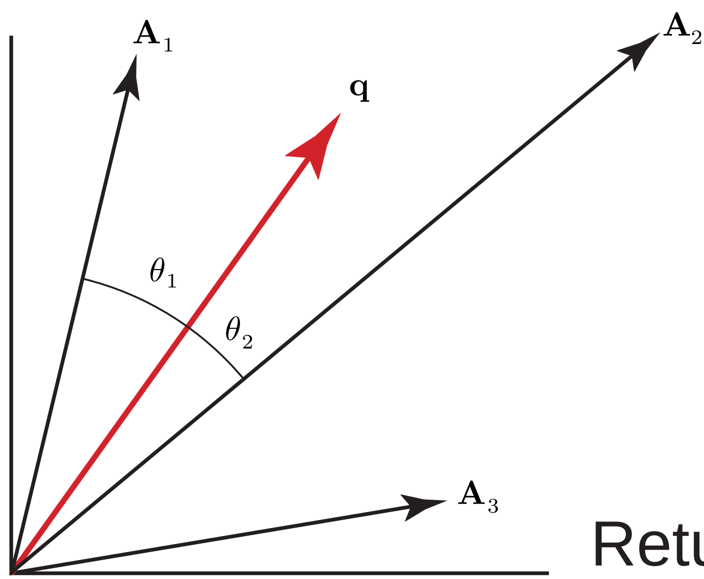

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Return Document i to user when $\delta_i \geq tol$

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- If the term $bank$ occurs once in Doc $\mathbf{1}$ but twice in Doc $\mathbf{2}$, and if $\|\mathbf{A_1}\| \approx \|\mathbf{A_2}\|$, then a query containing only $bank$ produces $\delta_\mathbf{2} \approx \mathbf{2}\delta_\mathbf{1}$ (i.e., Doc $\mathbf{2}$ is rated twice as relevant as Doc $\mathbf{1}$).

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- Set $q_i = \begin{cases} \log(n/\nu_i) & \text{if } \nu_i \neq 0 \\ 0 & \text{if } \nu_i = 0 \end{cases}$ (other weights also possible)

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Variation in Indexing Conventions

- No two people index documents the same way
- Computer indexing is inexact and can be unpredictable

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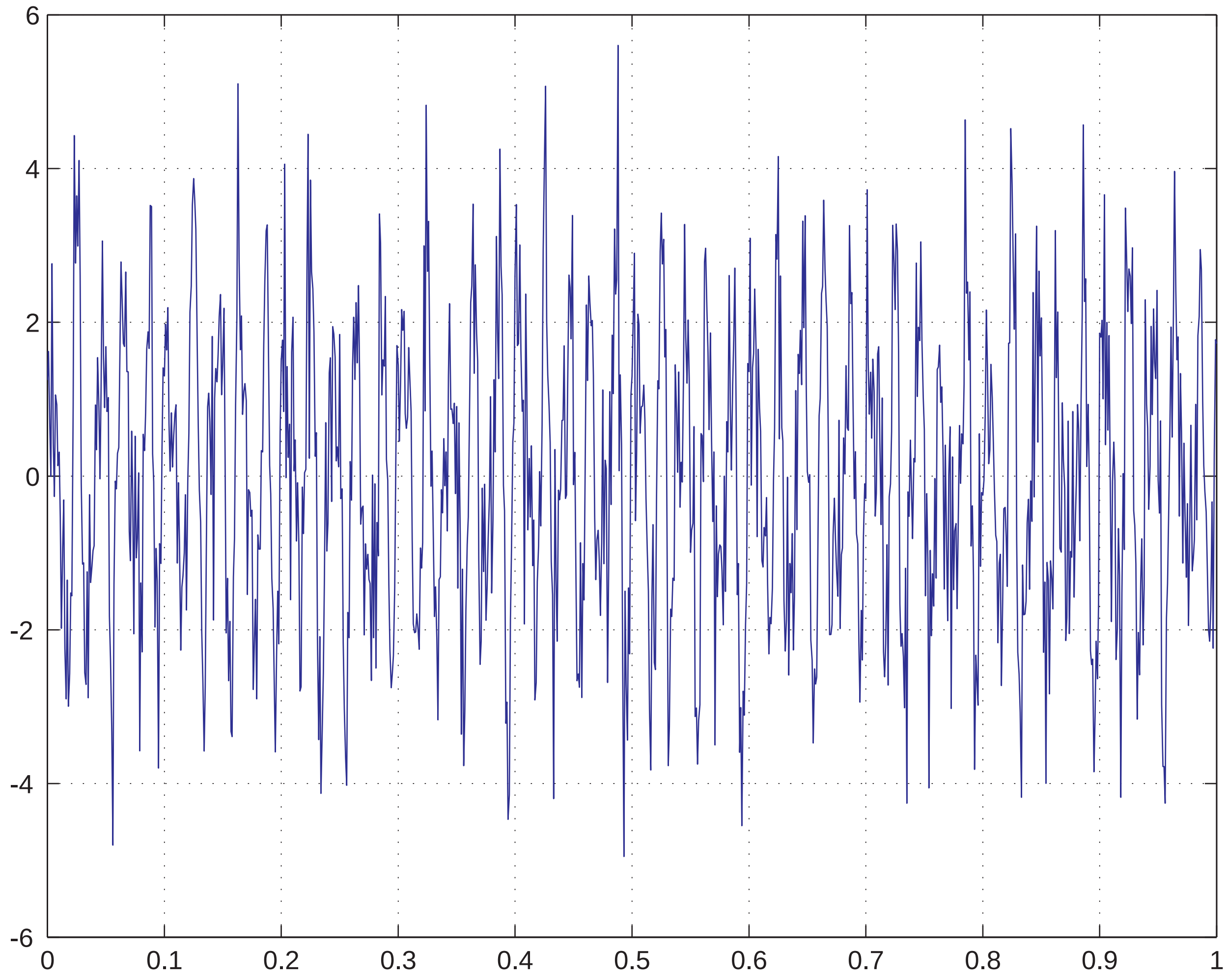

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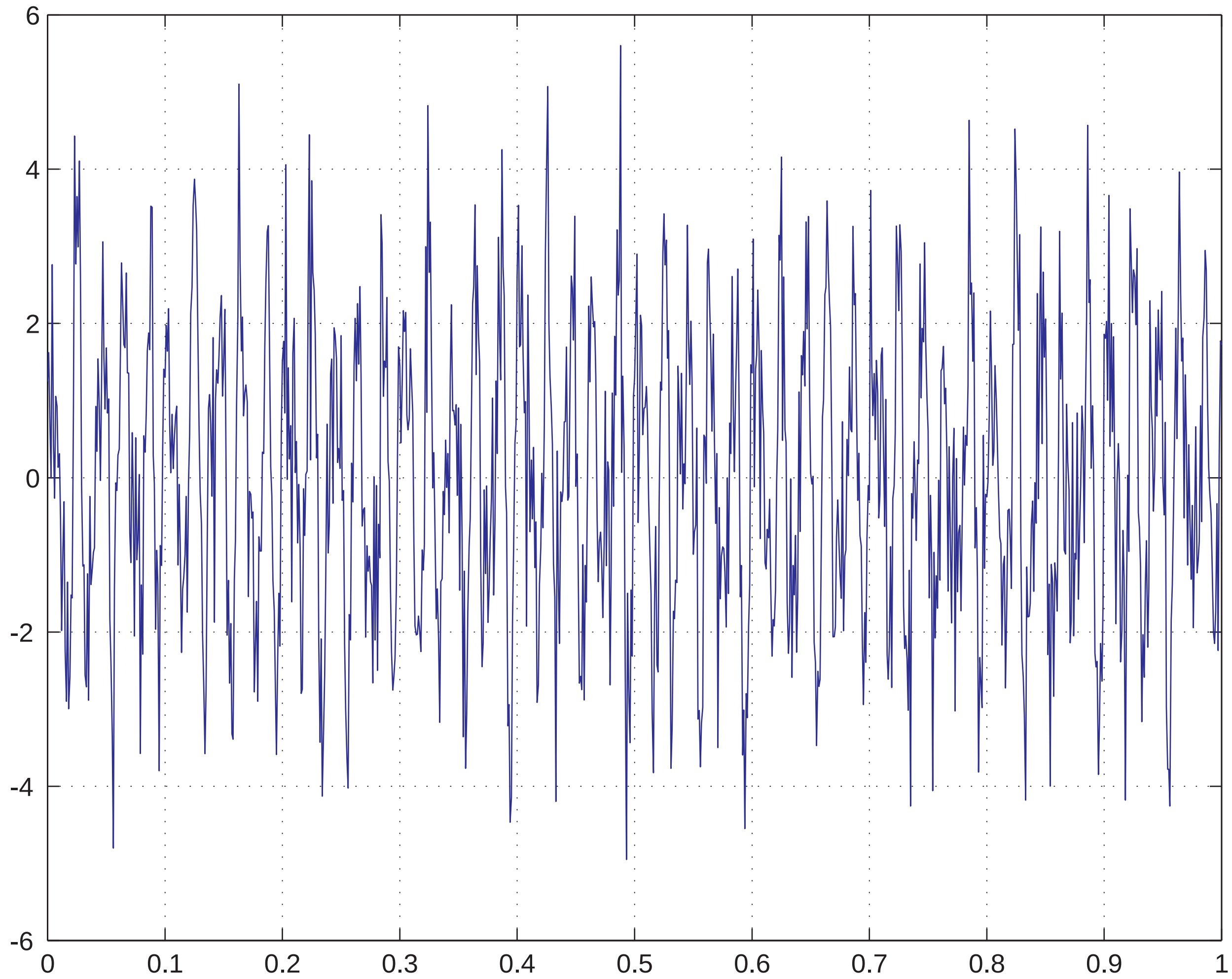

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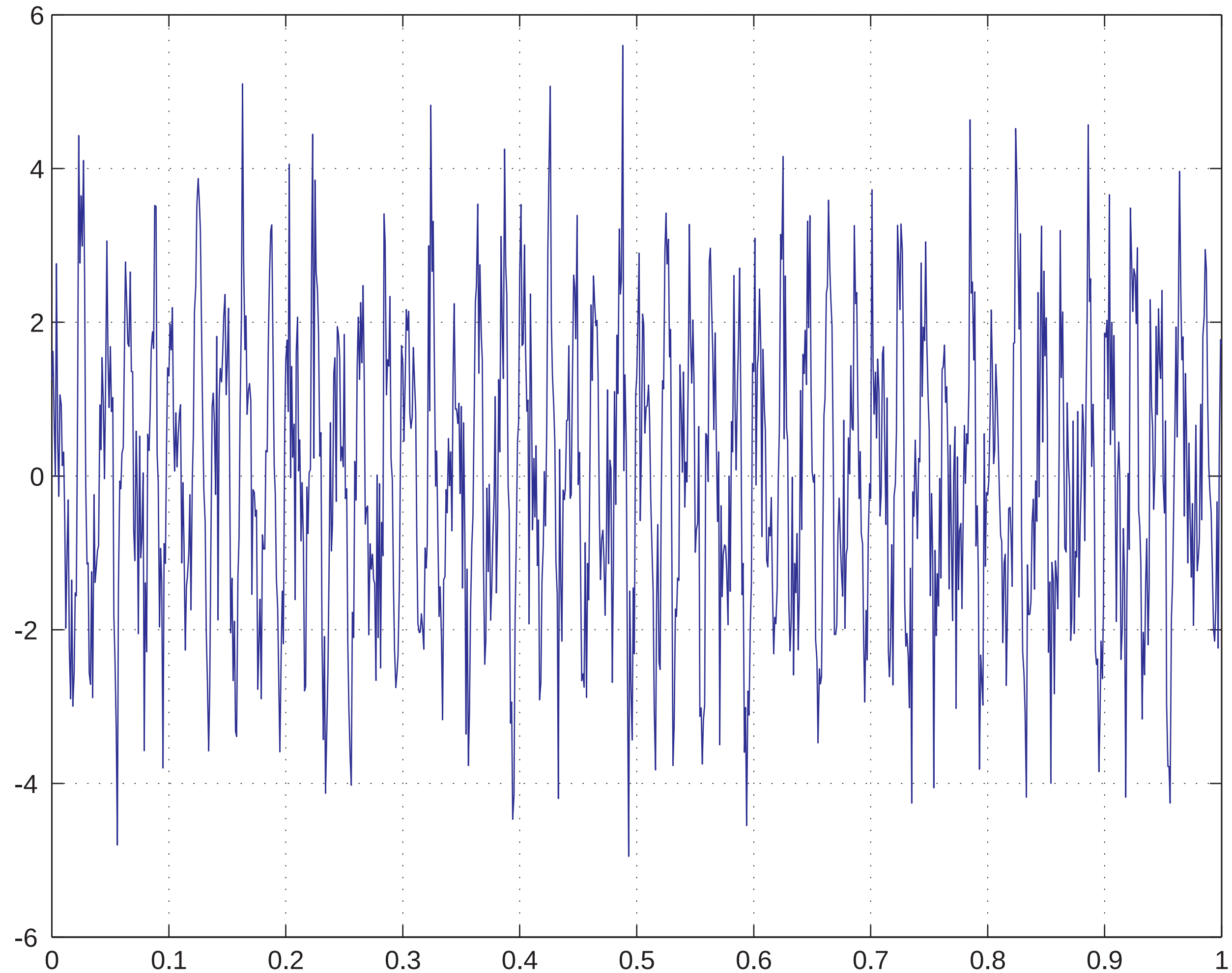

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Change Of Coordinates

New Basis $\mathcal{B} = \{\mathbf{W}_0, \mathbf{W}_1, \ldots, \mathbf{W}_{n-1}\}$

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  — Find y_k so that $\mathbf{x} = \sum y_k W_k$ (Fourier expansion if $\mathcal{B}$ o.n.)
  — $y_k = \langle \mathbf{W}_k | \mathbf{x} \rangle$ = amount of $\mathbf{x}$ in direction of W_k (if $\mathcal{B}$ o.n.)
  — $\mathbf{x} = \mathbf{W}\mathbf{y}$ where $\mathbf{W} = (W_0 \,|\, W_1 \,|\, \cdots \,|\, W_{n-1})$
  — $\mathbf{y} = \mathbf{W}^{-1}\mathbf{x}$ ($\mathbf{y}=\mathbf{W}^*\mathbf{x}$ if $\mathcal{B}$ o.n.)

Oscillatory

- $$\mathbf{W} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega & \omega^2 & \cdots & \omega^{n-1} \\ 1 & \omega^2 & \omega^4 & \cdots & \omega^{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{n-1} & \omega^{n-2} & \cdots & \omega \end{bmatrix}_{n\times n} \qquad \omega = e^{2\pi i/n}$$

Change Of Coordinates

New Basis $\mathcal{B} = \{\mathbf{W}_0, \mathbf{W}_1, \ldots, \mathbf{W}_{n-1}\}$

- Find coordinates of $\mathbf{x}$ with respect to $\mathcal{B}$
  - Find y_k so that $\mathbf{x} = \sum y_k W_k$ (Fourier expansion if $\mathcal{B}$ o.n.)
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  $t = 0, 1/n, 2/n, \ldots$

Change Of Coordinates

New Basis $\mathcal{B} = \{\mathbf{W}_0, \mathbf{W}_1, \ldots, \mathbf{W}_{n-1}\}$

- Find coordinates of $\mathbf{x}$ with respect to $\mathcal{B}$
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  $t = 0, 1/n, 2/n, \ldots$

- $W_k + W_{n-k} = \cos 2\pi k\mathbf{t}$

Change Of Coordinates

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- Find coordinates of $\mathbf{x}$ with respect to $\mathcal{B}$
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  $t = 0, 1/n, 2/n, \ldots$

- $W_k + W_{n-k} = \cos 2\pi k \mathbf{t}$
- $W_k - W_{n-k} = i \sin 2\pi k \mathbf{t}$ $\quad (0 < k < n)$

Making The Change

Making The Change

Recall

- $\mathbf{x} = \sum y_k W_k = \mathbf{W}\mathbf{y}$

Making The Change

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- $\mathbf{x} = \sum y_k W_k = \mathbf{W}\mathbf{y}$
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Making The Change

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- $\mathbf{x} = \sum y_k W_k = \mathbf{W}\mathbf{y}$
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$\mathbf{W}^{-1} = (4/n)\overline{\mathbf{W}}$ = Discrete Fourier Transform

Making The Change

Recall

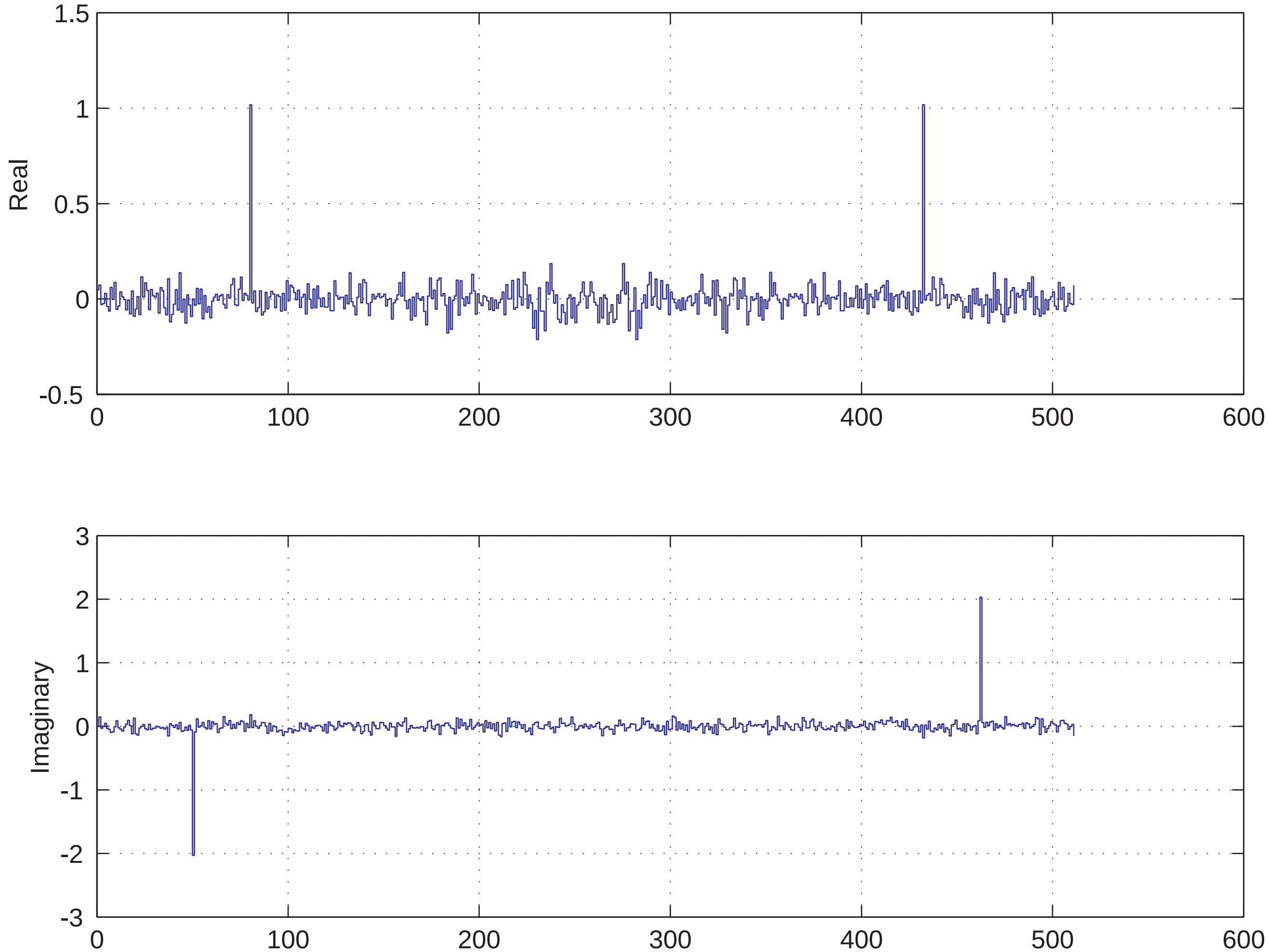
- $\mathbf{x} = \sum y_k W_k = \mathbf{Wy}$
- $\mathbf{y} = \mathbf{W}^{-1}\mathbf{x}$

$\mathbf{W}^{-1} = (4/n)\overline{\mathbf{W}}$ = Discrete Fourier Transform

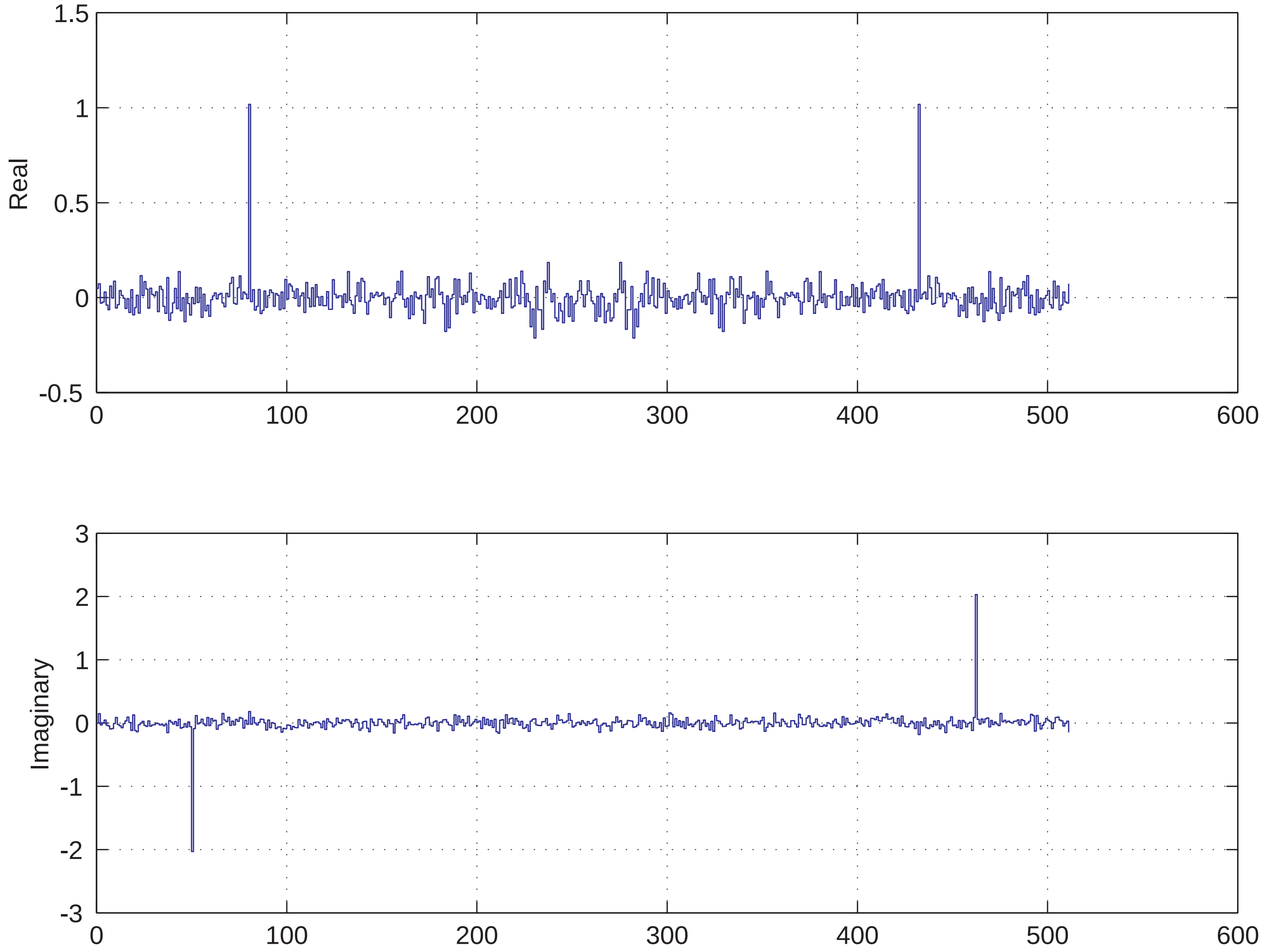
$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ y_{n-1} \end{bmatrix} = \frac{\mathbf{2}}{n} \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & \cdots & \mathbf{1} \\ \mathbf{1} & \xi & \xi^{\mathbf{2}} & \cdots & \xi^{n-\mathbf{1}} \\ \mathbf{1} & \xi^{\mathbf{2}} & \xi^{\mathbf{4}} & \cdots & \xi^{n-\mathbf{2}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{1} & \xi^{n-\mathbf{1}} & \xi^{n-\mathbf{2}} & \cdots & \xi \end{bmatrix} \begin{bmatrix} x_{\mathbf{0}} \\ x_{\mathbf{1}} \\ x_{\mathbf{2}} \\ \vdots \\ x_{n-\mathbf{1}} \end{bmatrix}$$

$$\xi = \mathrm{e}^{-\mathbf{2}\pi \mathrm{i}/n} = \overline{\omega}$$

The New Coordinates

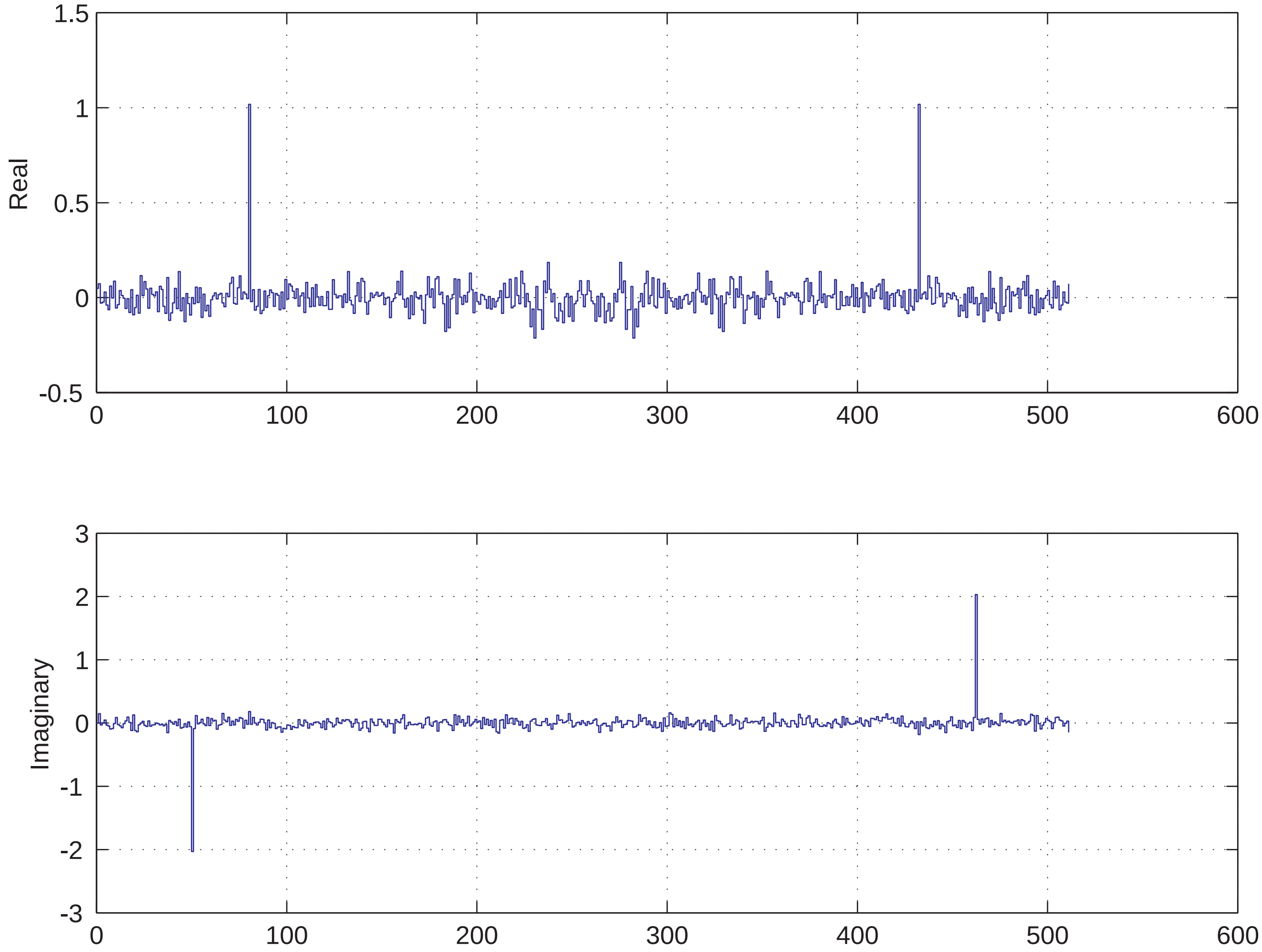


The New Coordinates



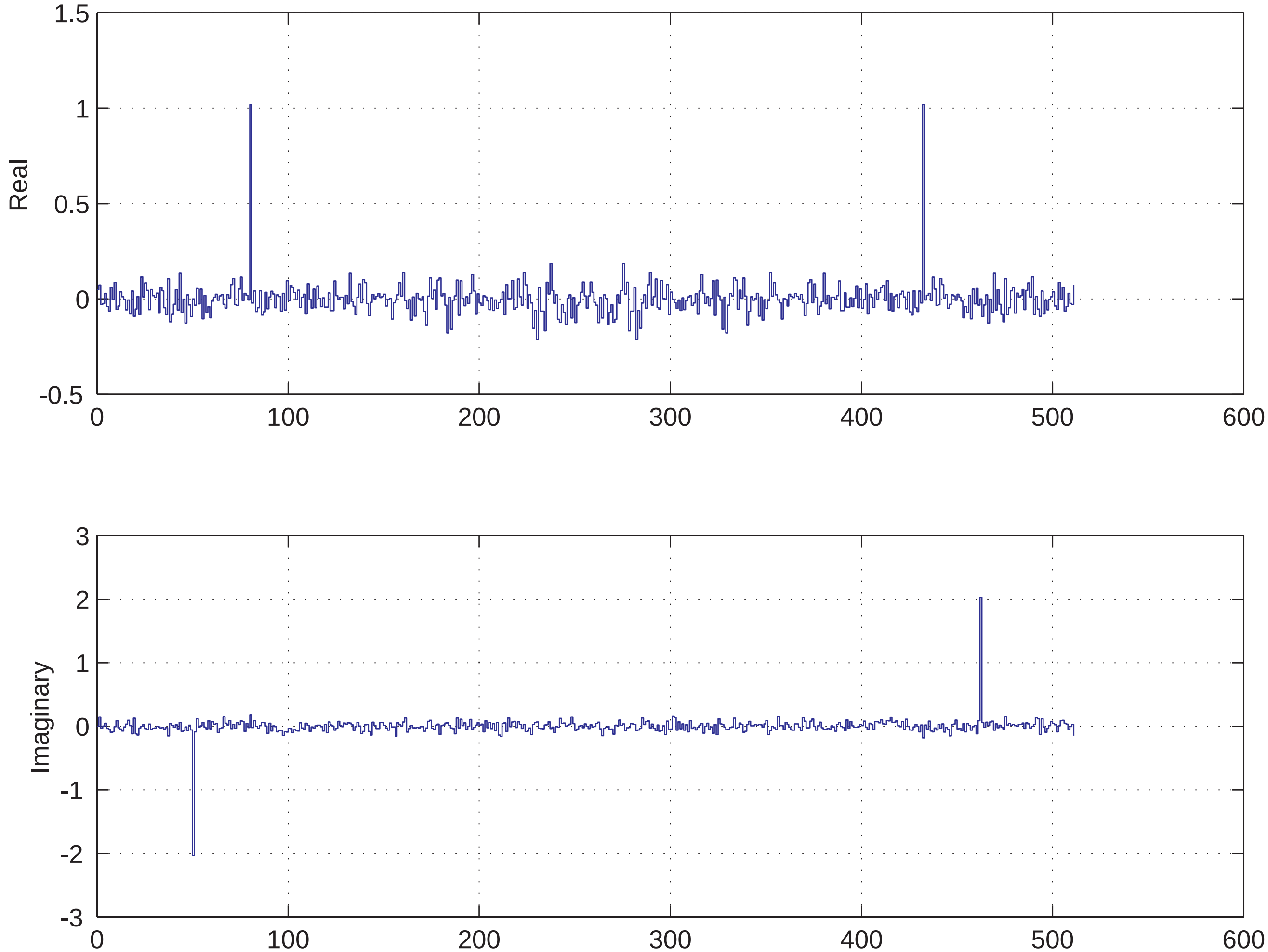
- Only 4 are significant: $y_{80} = y_{432} = 1$

The New Coordinates



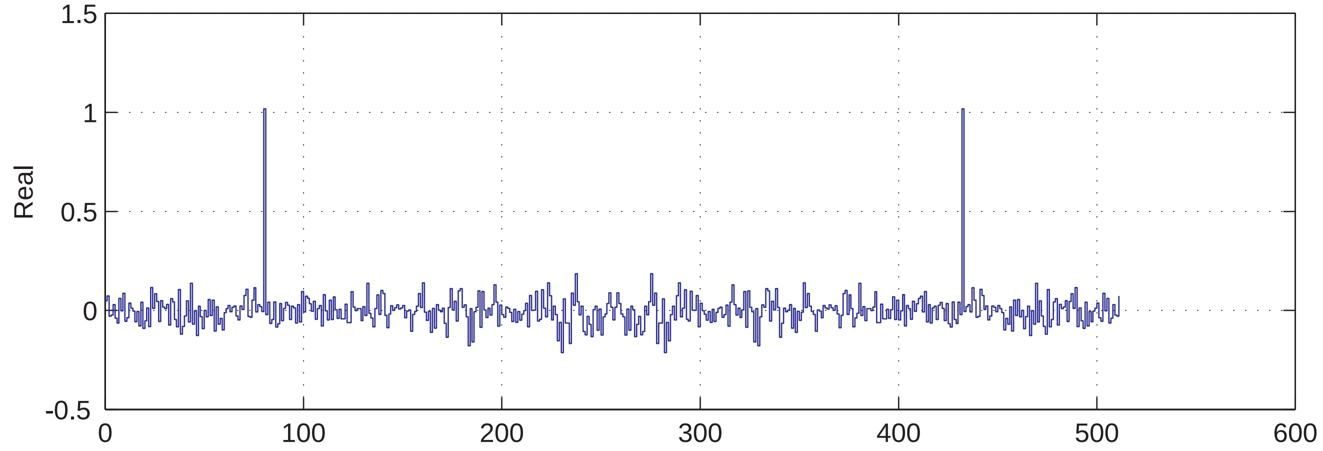
- Only 4 are significant: $y_{80} = y_{432} = 1$ and $y_{50} = -2\mathrm{i} = -y_{462}$

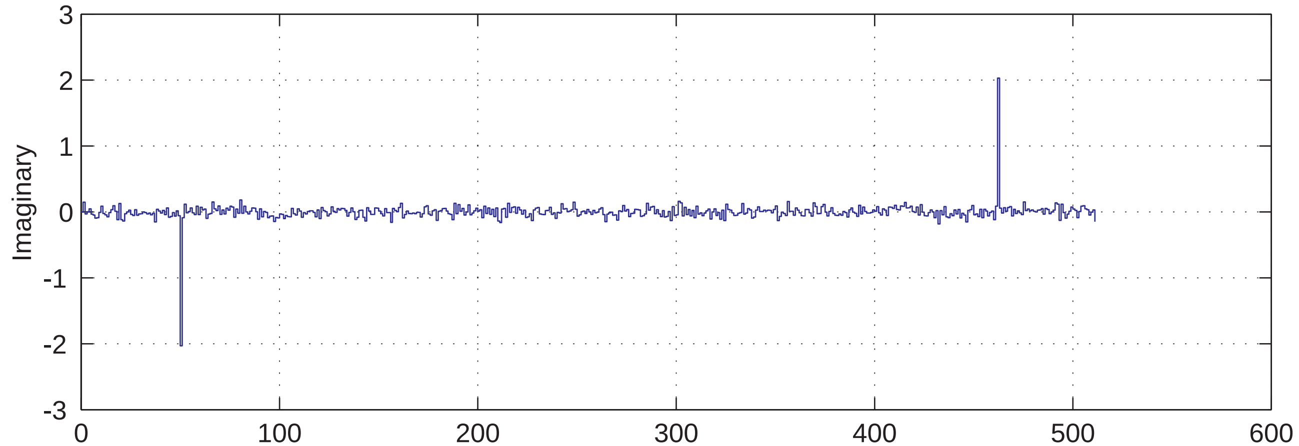
The New Coordinates



- Only 4 are significant: $y_{80} = y_{432} = 1$ and $y_{50} = -2\mathrm{i} = -y_{462}$
- $\mathbf{x} = \sum y_k W_k = 1W_{80} + 1W_{432} - 2\mathrm{i}W_{50} + 2\mathrm{i}W_{462} + \sum \varepsilon_j W_j$

The New Coordinates





- Only 4 are significant: $y_{80} = y_{432} = 1$ and $y_{50} = -\mathbf{2}\mathrm{i} = -y_{462}$
- $\mathbf{x} = \sum y_k W_k = \mathbf{1}W_{\mathbf{80}} + \mathbf{1}W_{\mathbf{432}} - \mathbf{2}\mathrm{i}W_{\mathbf{50}} + \mathbf{2}\mathrm{i}W_{\mathbf{462}} + \sum \varepsilon_j W_j$
- Small components (noise) are nondirectional

Drop Small Coordinates

- $\mathbf{x} = \sum y_k W_k = \mathbf{1} W_{\mathbf{80}} + \mathbf{1} W_{\mathbf{432}} - \mathbf{2}\mathrm{i} W_{\mathbf{50}} + \mathbf{2}\mathrm{i} W_{\mathbf{462}} + \sum \varepsilon_j W_j$

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- $\mathbf{x} = \sum y_k W_k = \mathbf{1} W_{\mathbf{80}} + \mathbf{1} W_{\mathbf{432}} - \mathbf{2}\mathrm{i} W_{\mathbf{50}} + \mathbf{2}\mathrm{i} W_{\mathbf{462}} + \sum \varepsilon_j W_j$
- $\widetilde{\mathbf{x}} = (W_{\mathbf{80}} + W_{\mathbf{432}}) - \mathbf{2}\mathrm{i}(W_{\mathbf{50}} - W_{\mathbf{462}})$

Drop Small Coordinates

- $\mathbf{x} = \sum y_k W_k = \mathbf{1} W_{\mathbf{80}} + \mathbf{1} W_{\mathbf{432}} - \mathbf{2}\mathsf{i} W_{\mathbf{50}} + \mathbf{2}\mathsf{i} W_{\mathbf{462}} + \sum \varepsilon_j W_j$
- $\widetilde{\mathbf{x}} = (W_{\mathbf{80}} + W_{\mathbf{432}}) - \mathbf{2}\mathsf{i}(W_{\mathbf{50}} - W_{\mathbf{462}})$
- $n = \mathbf{512}$
- $\widetilde{\mathbf{x}} = (W_{\mathbf{80}} + W_{n-\mathbf{80}}) - \mathbf{2}\mathsf{i}(W_{\mathbf{50}} - W_{n-\mathbf{50}})$

Drop Small Coordinates

- $\mathbf{x} = \sum y_k W_k = \mathbf{1} W_{\mathbf{80}} + \mathbf{1} W_{\mathbf{432}} - \mathbf{2}\mathrm{i} W_{\mathbf{50}} + \mathbf{2}\mathrm{i} W_{\mathbf{462}} + \sum \varepsilon_j W_j$
- $\widetilde{\mathbf{x}} = (W_{\mathbf{80}} + W_{\mathbf{432}}) - \mathbf{2}\mathrm{i}(W_{\mathbf{50}} - W_{\mathbf{462}})$
- $n = \mathbf{512}$
- $\widetilde{\mathbf{x}} = (W_{\mathbf{80}} + W_{n-\mathbf{80}}) - \mathbf{2}\mathrm{i}(W_{\mathbf{50}} - W_{n-\mathbf{50}})$ Compressed $(\mathbf{512} \to \mathbf{4})$

Drop Small Coordinates

- $\mathbf{x} = \sum y_k W_k = 1W_{80} + 1W_{432} - 2\mathrm{i}W_{50} + 2\mathrm{i}W_{462} + \sum \varepsilon_j W_j$
- $\widetilde{\mathbf{x}} = (W_{80} + W_{432}) - 2\mathrm{i}(W_{50} - W_{462})$
- $n = 512$
- $\widetilde{\mathbf{x}} = (W_{80} + W_{n-80}) - 2\mathrm{i}(W_{50} - W_{n-50})$ Compressed $(512 \to 4)$
  - $W_k + W_{n-k} = \cos 2\pi k\mathbf{t}$
  - $W_k - W_{n-k} = \mathrm{i}\sin 2\pi k\mathbf{t}$

Drop Small Coordinates

- $\mathbf{x} = \sum y_k W_k = 1W_{80} + 1W_{432} - 2\mathrm{i}W_{50} + 2\mathrm{i}W_{462} + \sum \varepsilon_j W_j$
- $\widetilde{\mathbf{x}} = (W_{80} + W_{432}) - 2\mathrm{i}(W_{50} - W_{462})$
- $n = 512$
- $\widetilde{\mathbf{x}} = (W_{80} + W_{n-80}) - 2\mathrm{i}(W_{50} - W_{n-50})$ Compressed $(512 \to 4)$
  - $W_k + W_{n-k} = \cos 2\pi k\mathbf{t}$
  - $W_k - W_{n-k} = \mathrm{i}\,\sin 2\pi k\mathbf{t}$
- $\widetilde{\mathbf{x}} = \cos 2\pi 80\mathbf{t} + 2\sin 2\pi 50\mathbf{t}$

Drop Small Coordinates

- $\mathbf{x} = \sum y_k W_k = 1W_{80} + 1W_{432} - 2\mathrm{i}W_{50} + 2\mathrm{i}W_{462} + \sum \varepsilon_j W_j$
- $\widetilde{\mathbf{x}} = (W_{80} + W_{432}) - 2\mathrm{i}(W_{50} - W_{462})$
- $n = 512$
- $\widetilde{\mathbf{x}} = (W_{80} + W_{n-80}) - 2\mathrm{i}(W_{50} - W_{n-50})$ Compressed $(512 \to 4)$
  - $W_k + W_{n-k} = \cos 2\pi k\mathbf{t}$
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- $\widetilde{\mathbf{x}} = \cos 2\pi 80\mathbf{t} + 2\sin 2\pi 50\mathbf{t}$ Cleaned

Drop Small Coordinates

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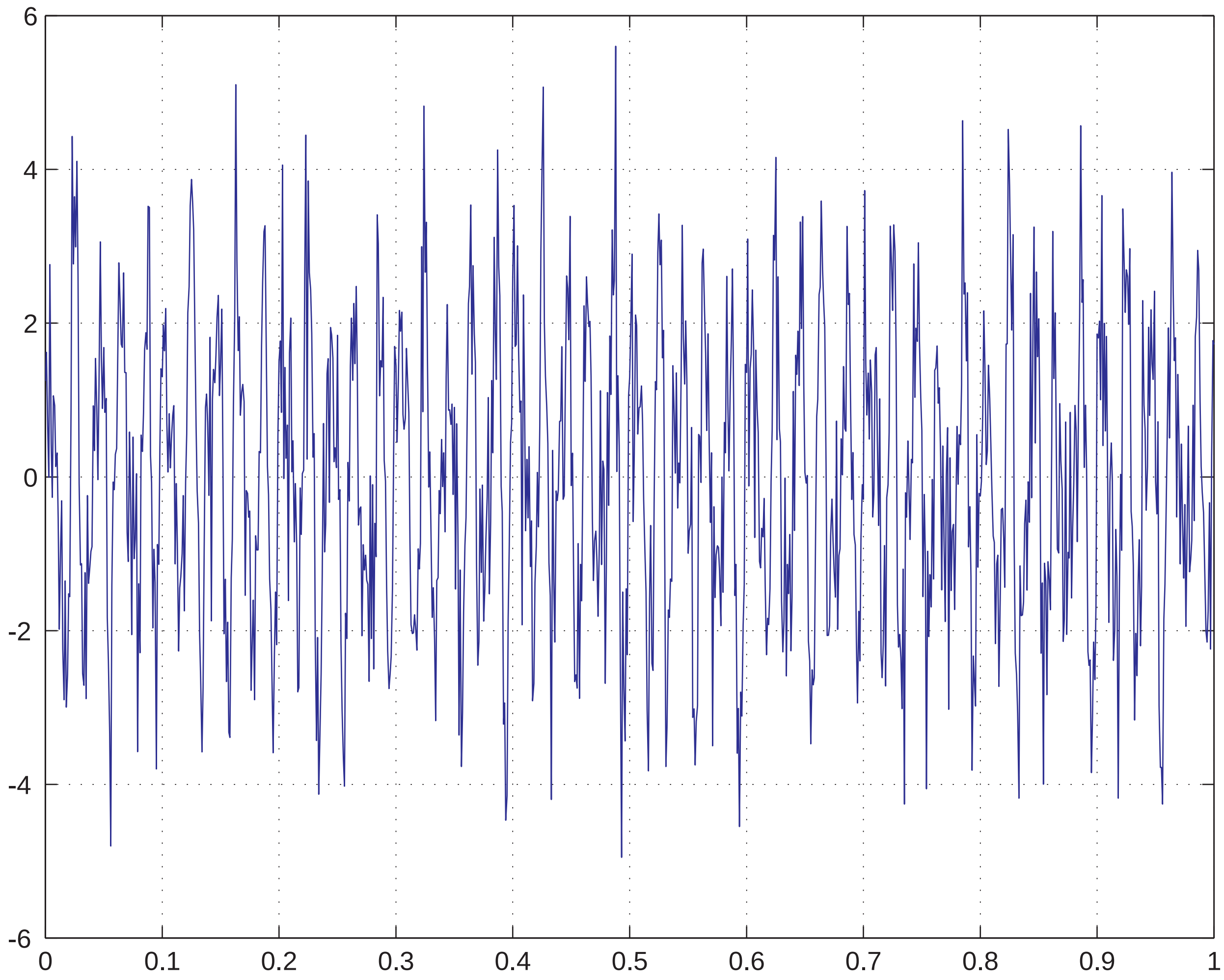
- $n = 512$
- $\widetilde{\mathbf{x}} = (W_{80} + W_{n-80}) - 2\mathrm{i}(W_{50} - W_{n-50})$ Compressed $(512 \to 4)$
  - $W_k + W_{n-k} = \cos 2\pi k\mathbf{t}$
  - $W_k - W_{n-k} = \mathrm{i}\ \sin 2\pi k\mathbf{t}$
- $\widetilde{\mathbf{x}} = \cos 2\pi 80\mathbf{t} + 2 \sin 2\pi 50\mathbf{t}$ Cleaned
- $\mathbf{x} = \cos 2\pi 80\mathbf{t} + 2 \sin 2\pi 50\mathbf{t} + \text{noise}$

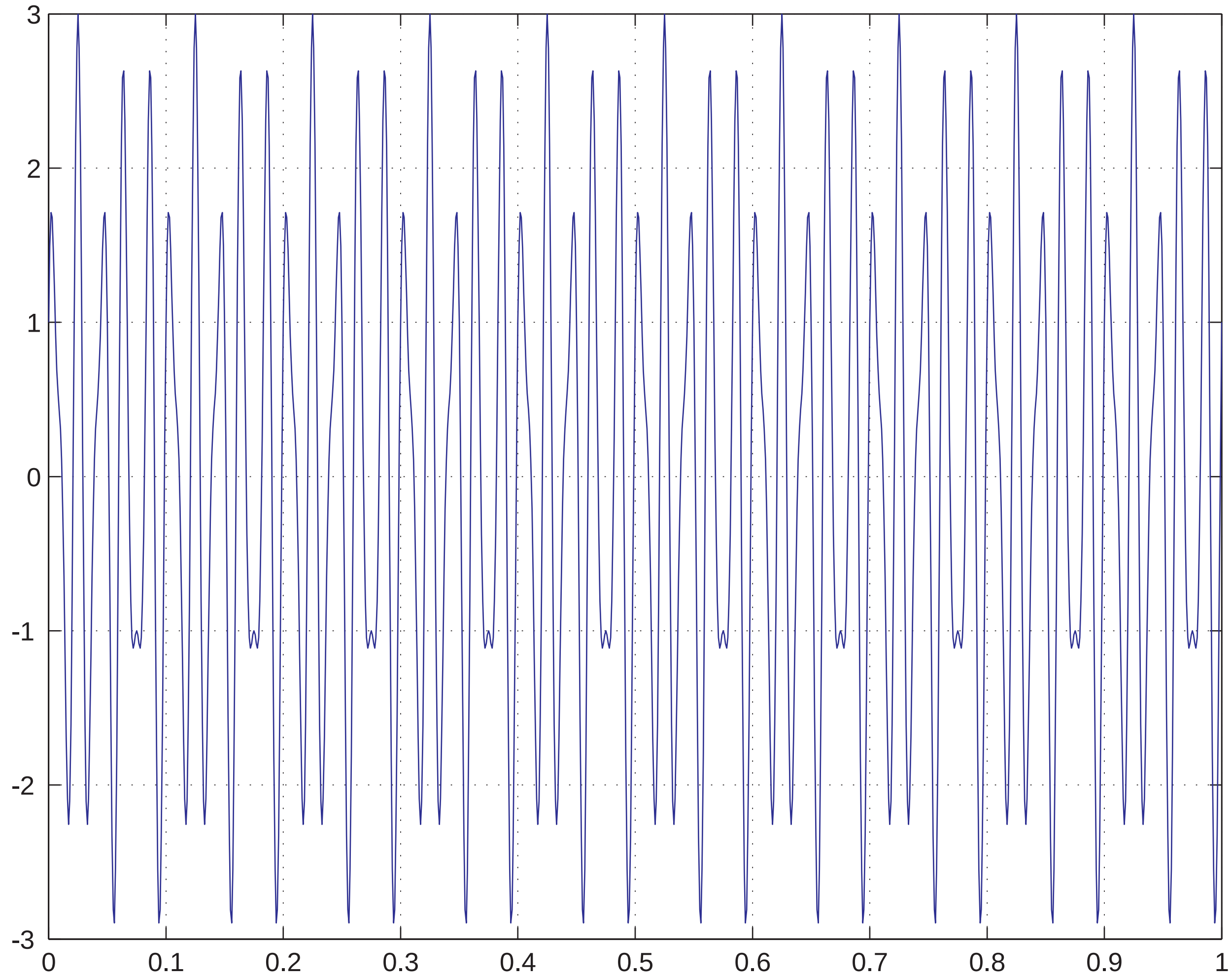
Original Data

$$\mathbf{x} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_{510} \\ x_{511} \end{bmatrix}$$



Cleaned & Compressed Data

$$\widetilde{\mathbf{x}} = \mathbf{x} - \text{noise} = (W_{80} + W_{432}) - 2i(W_{50} - W_{462})$$



$$\cos 2\pi 80\mathbf{t} + 2\sin 2\pi 50\mathbf{t}$$

The DFT Game

Matrix–Vector Product

$$\mathbf{y} = \frac{2}{n}\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \xi & \xi^2 & \cdots & \xi^{n-1} \\ 1 & \xi^2 & \xi^4 & \cdots & \xi^{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \xi^{n-1} & \xi^{n-2} & \cdots & \xi \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_{n-1} \end{bmatrix} \qquad \xi = \mathrm{e}^{-2\pi\mathrm{i}/n}$$

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Simple in Theory, But $\cdots$

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- Must do it $FAST!$

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Need For Speed $\Longrightarrow$ Matrix Factorizations $\Longrightarrow$ FFT

The DFT Game

Matrix–Vector Product

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- $\mathbf{F}_n = \mathbf{B}_n(\mathbf{I}_2 \otimes \mathbf{F}_{n/2})\mathbf{P}_n \qquad \mathbf{B}_n = \begin{bmatrix} \mathbf{I}_{n/2} & \mathbf{D}_{n/2} \\ \mathbf{I}_{n/2} & -\mathbf{D}_{n/2} \end{bmatrix} \qquad \mathbf{D}_{n/2} = \begin{bmatrix} 1 & & & \\ & \xi & & \\ & & \xi^2 & \\ & & & \ddots \end{bmatrix}$

The DFT Game

Matrix–Vector Product

$$\mathbf{y} = \frac{2}{n}\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \xi & \xi^2 & \cdots & \xi^{n-1} \\ 1 & \xi^2 & \xi^4 & \cdots & \xi^{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \xi^{n-1} & \xi^{n-2} & \cdots & \xi \end{bmatrix}\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_{n-1} \end{bmatrix} \qquad \xi = \mathrm{e}^{-2\pi \mathrm{i}/n}$$

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- FFT changes n^2 flop requirement into $(n/2)\log_2 n$

The DFT Game

Matrix–Vector Product

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- FFT changes n^2 flop requirement into $(n/2)\log_2 n$

"The most valuable numerical algorithm in our lifetime."
—G. Strang, Bulletin of the AMS, April, 1993.

Back To IR

Almost the Same Problem

- Reveal hidden patterns & evaluate $\mathbf{q}^T\mathbf{A}$ fast

Back To IR

Almost the Same Problem

- Reveal hidden patterns & evaluate $\mathbf{q}^T\mathbf{A}$ fast (clean & compress)

Back To IR

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Data is Now the Term-Doc Matrix in Standard Coordinates

- $\mathbf{A} = \sum_{i,j} a_{ij}\mathbf{E}_{ij} \qquad \mathbf{E}_{ij} = \mathbf{e}_i\mathbf{e}_j^T$

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Change Basis to $\mathcal{B} = \{\mathbf{Z}_1, \mathbf{Z}_2, \ldots\}$ That Can Squeeze & Clean

- $\mathbf{A} = \sum \sigma_i \mathbf{Z}_i$ (Fourier Expansion)

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- $\mathbf{A} = \sum \sigma_i \mathbf{Z}_i$ (Fourier Expansion)
- $\mathcal{B}$ o.n. $\Rightarrow \sigma_i = \langle \mathbf{Z}_i | \mathbf{A} \rangle$ = amount of $\mathbf{A}$ in direction of $\mathbf{Z}_i$

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Matrix Factorizations: $\mathbf{A} = \mathbf{URV}^T = \sum r_{ij}\mathbf{u}_i\mathbf{v}_j^T = \sum r_{ij}\mathbf{Z}_{ij}$

Back To IR

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Data is Now the Term-Doc Matrix in Standard Coordinates

- $\mathbf{A} = \sum_{i,j} a_{ij}\mathbf{E}_{ij}$ $\quad\quad$ $\mathbf{E}_{ij} = \mathbf{e}_i\mathbf{e}_j^T$

Change Basis to $\mathcal{B} = \{\mathbf{Z}_1, \mathbf{Z}_2, \ldots\}$ That Can Squeeze & Clean

- $\mathbf{A} = \sum \sigma_i\mathbf{Z}_i$ (Fourier Expansion)
- $\mathcal{B}$ o.n. $\Rightarrow \sigma_i = \langle \mathbf{Z}_i | \mathbf{A} \rangle$ = amount of $\mathbf{A}$ in direction of $\mathbf{Z}_i$

Matrix Factorizations: $\mathbf{A} = \mathbf{URV}^T = \sum r_{ij}\mathbf{u}_i\mathbf{v}_j^T = \sum r_{ij}\mathbf{Z}_{ij}$

- Represent data with as few directions $\mathbf{Z}_i$ as possible

Back To IR

Almost the Same Problem

- Reveal hidden patterns & evaluate $\mathbf{q}^T\mathbf{A}$ fast (clean & compress)

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- SVD $\Rightarrow \mathbf{R} = \begin{bmatrix} \sigma_1 & & & & \\ & \ddots & & & \\ & & \sigma_r & & \\ & & & 0 & \\ & & & & \ddots \\ & & & & & 0 \end{bmatrix} \Rightarrow \quad \mathbf{A} = \sum_{i=1}^r \sigma_i \mathbf{Z}_i, \qquad \langle \mathbf{Z}_i | \mathbf{Z}_j \rangle = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$

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- Compare query to each document
  - $\mathbf{q}^T \widetilde{\mathbf{A}} = \sum_{i=1}^{k} \sigma_i (\mathbf{q}^T \mathbf{u}_i) \widetilde{\mathbf{v}}_i^T = (\delta_1, \delta_2, \ldots, \delta_n)$

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How To Take Advantage of Link Structure ?

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Hubs & Authorities

(Jon Kleinberg 1998)

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- But if Yahoo! points to many places, the value of the link to P is diluted

PageRank

The Definition

- $r(P) = \sum_{P \in \mathcal{B}_P} \frac{r(P)}{|P|}$
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$$\ddots$$

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After Step j

- $\boldsymbol{\pi}_j^T = \left[r_j(P_1),\ r_j(P_2),\ \cdots,\ r_j(P_n) \right]$

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After Step j

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- $\boldsymbol{\pi}_{j+1}^T = \boldsymbol{\pi}_j^T \mathbf{P}$ where $p_{ij} = \begin{cases} 1/|P_i| & \text{if } i \rightarrow j \\ 0 & \text{otherwise} \end{cases}$
- PageRank $= \lim_{j\rightarrow\infty} \boldsymbol{\pi}_j^T = \boldsymbol{\pi}^T$ (provided limit exists)

It's A Markov Chain

- $\mathbf{P} = \left[p_{ij}\right]$ is a stochastic matrix (row sums = 1)
- Each $\boldsymbol{\pi}_j^T$ (and $\boldsymbol{\pi}^T$) is a probability vector $\left(\sum_i r_j(P_i) = 1\right)$
- $\boldsymbol{\pi}_{j+1}^T = \boldsymbol{\pi}_j^T \mathbf{P}$ is random walk on the graph defined by links

Random Surfer

Web Surfer Randomly Clicks On Links

(Back button not a link)

- Long-run proportion of time on page P_i is $\boldsymbol{\pi}_i$

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World's Largest Eigenvector Problem (C. Moler)

- Solve $\boldsymbol{\pi}^T = \boldsymbol{\pi}^T \mathbf{P}$ (stationary distribution vector)

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- Link structure of web is extremely dynamic

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- Old results don't help to restart (even if size doesn't change)
  - Cutoff phenomenon in random walks (P. Diaconis, 1996)

Report Card

FEATURES	LSI	LINK ANALYSIS

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Reveals Hidden Patterns		

Report Card

FEATURES	LSI	LINK ANALYSIS
Reveals Hidden Patterns	A	

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Scales Up		

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Hybrid Approach

The Idea

- Use link structure to define measure of page (doc) contiguity
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3. Return $\mathcal{P}$ together with those $\mathcal{P} \rightarrow P_i, P_j, P_k, P_l, \ldots$

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What's the $\textit{``distance''}$ **from** D_i **to** D_j **?**

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Directed Link Structure $\Longrightarrow$ **Nonsymmetric Metric**