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Outline of PageRank

- \[ G = \alpha[H + (1/n)ae^T] + (1 - \alpha)ev^T \]
  - \( H \) is a hyperlink matrix,
  - \( 0 < \alpha < 1 \), \( e \) is a vector of 1’s
  - \( a_i = 0 \) if \( H_i^T \) is nonzero and \( a_i = 1 \) otherwise,
  - \( v > 0 \) is a probability distribution vector.

- The vector containing the ratings of each web page is \( \pi \) such that
  \[ \pi^T = \pi^T G \]

Use rating scores in \( \pi \) to rank web pages.
Generalizing PageRank, GeM

\[ G = \alpha_0 S_0 + \alpha_1 S_1 + \ldots + \alpha_p S_p \]

\[ \text{Each } S_i \text{ is stochastic (derived from sports statistics),} \]
\[ 0 \leq \alpha_i \leq 1 \text{ and } \sum \alpha_i = 1. \]

\[ \text{The vector } \pi \text{ contains the rating scores of each team, such that} \]
\[ \pi^T = \pi^T G \]

Use rating scores in \( \pi \) to rank teams.
Gem1 - Basic model

\[ G = \alpha [H + au^T] + (1 - \alpha)ee^T \]

- \( 0 < \alpha < 1 \),
- \( H \) is based on game scores,
- \( a_i = 0 \) if row \( i \) of \( H \) is nonzero and \( a_i = 1 \) otherwise,
- \( u \) is a probability distribution.
Gem2 - Feature vectors model

\[ G = \alpha_0[H + au^T] + \alpha_1 ev_1^T + \ldots + \alpha_p ev_p^T \]

- \( 0 \leq \alpha_i \leq 1, \Sigma \alpha_i = 1 \).
- \( H \) is derived using game scores,
- \( a_i = 0 \) if row \( i \) of \( H \) is nonzero and \( a_i = 1 \) otherwise,
- \( u \) and \( v_i \) are probability distribution vectors.
- Compute \( v_i \) using nonnegative matrix factorization of a matrix containing all of the statistics.
Gem3 - Offense-defense model

\[ G = \alpha_0[H + au^T] + \alpha_1eo^T + \alpha_2ed^T \]

- \( 0 \leq \alpha_i \leq 1, \Sigma\alpha_i = 1, \)
- \( H \) is derived using game scores.
- \( a_i = 0 \) if row \( i \) of \( H \) is nonzero and \( a_i = 1 \) otherwise,
- \( u \) is a probability distribution vector.
- Compute offense vector \( o \) and defense vector \( d \) using modified HITS.
Gem4 - Feature matrices model

\[ G = \alpha_0[H_0 + a_0 u_0^T] + \ldots + \alpha_p[H_p + a_p u_p^T] \]

- \( 0 \leq \alpha \leq 1 \), \( \Sigma \alpha_i = 1 \),
- \( H_j \) is derived using statistic \( j \) (e.g. scores, yards, etc.),
- \( a_{ij} = 0 \) if row \( i \) of \( H_j \) is nonzero and \( a_{ij} = 1 \) otherwise,
- \( u_j \) is a probability distribution vector.
Game Prediction Results

Results of NFL game predictions.