P. P. I. Factors

P lemmons

Positive

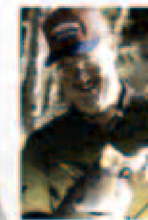
Influence Factor’s

(Relative To Me)

Model A Bomber

BOMBER

ONLY $3.59



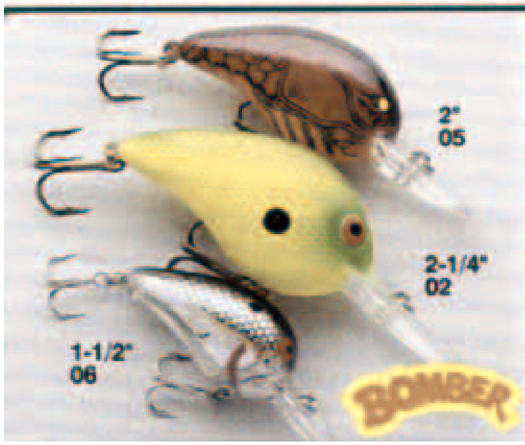
"There are lures that everyone simply has to have. The Bomber® Model A's are in that category. If you want to catch bass on crankbaits, you've got to have Model A's."
Mark Davis
1998 B.A.S.S. Angler-of-the-Year

Tournament Winning Lures!

Model "A"® The Model "A"® is a little slimmer, a little trimmer, a little faster wiggler, and a little deeper diver than the ordinary crankbait. Most importantly, its construction is different. We've made an integral, molded bill-to-body design for extra strength and true tuning. ***Order by color code:*** Dark Green Crawdad(02), Dark Brown Crawdad(04), Red Crawdad(05), Chart. Crawdad(07), Firetiger(15), Tenn. Shad(22), Light Baby Bass/Org. Belly(32), G-Finish Bengal Fire(50), G-Finish Dark Brown Crawdad(51). NEW COLORS: Lemon Lime/Org. Belly(08), Brown on Yellow/Org. Belly(11), White(12), Baby Striper(13), Chrome/Blue Back/Black(17), Silver Shad(18), Bream/Orange Belly(19), Chrome/Black Back(20), Chrome/Black(24), Dull Flour/Yellow(36), Dark Baby Bass/Org. Belly(39).

Item	Model	Lgth.	Weight (oz.)	Dives	Hook Size
28-255-580*	2A	2-1/8"	5/16	3'-5'	#6
28-255-637**	5A	1-7/8"	1/4	4'-6'	#6
28-255-645†	6A	2-1/8"	3/8	8'-10'	#6
28-255-696††	7A	2-5/8"	1/2	8'-10'	#4

*Not Available in 20, 39, 50, 51
**Not available in 08, 13, 17, 50, 51
†Not available in 08, 11, 12, 13, 17, 18, 19, 20, 24, 36, 39
††Not available 08, 11, 12, 13, 17, 19, 20, 50, 51

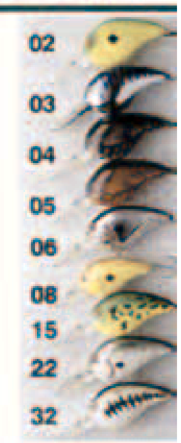


Fat "A"™ The Fat "A"™ series is designed to deliver a buoyancy characteristic equal to or greater than wooden crankbaits. The "fatter," more buoyant body design also provides more internal space for additional ultra-sound rattles, making the Fat "A"™ series the loudest among similarly sized crankbaits. The molded-in diving lip ensures "true-running" performance and its length, in reference to body size, makes the Fat "A"™ series one of the most snag-free lures Bomber® offers. ***Order by color code:*** Dark Brown Crawdad(04), Red Crawdad(05), Firetiger(15), Tennessee Shad(22), Baby Bass/Orange Belly(32). NEW COLORS: Lemon Lime/Org. Belly(02), Chrome/Blue Back/Black(03), Silver Flash(06), Brown on Yellow/Orange Back(08).

Item	Model	Lgth.	Wt.(oz.)	Dives to	Hook Size
28-255-665*	4F	1-1/2"	1/5	4'-6'	#6
28-255-680**	5F	2"	3/8	6'-8'	#5
28-255-670†	6F	2-1/4"	5/8	8'-10'	#3

Bass Pro Price
3.59 YOUR CHOICE

*Not available in 02, 03, 05, 08 **Not available in 02, 03 †Not available in 08



Nonnegative Matrices

In The Mathematical Sciences

(With Avi Berman)

Light Perron–Frobenius

Uncoupling The Perron Vector

- $\mathbf{A}_{n \times n}$ Nonnegative & Irreducible
- $\mathbf{Ax} = \rho\mathbf{x}$ ρ = Spec Radius $\mathbf{x} > \mathbf{0}$ $\sum x_i = 1$
- Partition $\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \cdots & \mathbf{A}_{1k} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \cdots & \mathbf{A}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{k1} & \mathbf{A}_{k2} & \cdots & \mathbf{A}_{kk} \end{bmatrix}$ $\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_k \end{bmatrix}$

The Goal

- Determine each $\mathbf{x}_i$ independently
- Each $\mathbf{x}_i$ should solve a P–F problem of size $\sim \mathbf{A}_{ii}$

Perron Complementation

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}$$

- $\mathbf{P}_{11} = \mathbf{A}_{11} + \mathbf{A}_{12}(\rho\mathbf{I} - \mathbf{A}_{22})^{-1}\mathbf{A}_{21}$
- $\mathbf{P}_{22} = \mathbf{A}_{22} + \mathbf{A}_{21}(\rho\mathbf{I} - \mathbf{A}_{11})^{-1}\mathbf{A}_{12}$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \star & \mathbf{A}_{1i} & \star & \mathbf{A}_{1k} \\ \star & \star & | & \star & \star \\ \mathbf{A}_{i1} & - & \mathbf{A}_{ii} & - & \mathbf{A}_{ik} \\ \star & \star & | & \star & \star \\ \mathbf{A}_{k1} & \star & \mathbf{A}_{ki} & \star & \mathbf{A}_{kk} \end{bmatrix}$$

- $\mathbf{P}_{ii} = \mathbf{A}_{ii} + \mathbf{A}_{i\star}(\rho I - \overline{\mathbf{A}_{ii}})^{-1}\mathbf{A}_{\star i}$

Inheritance

(Fun with M–matrices)

Each $\mathbf{P}_{ii} = \mathbf{A}_{ii} + \mathbf{A}_{i\star}(\rho\mathbf{I} - \overline{\mathbf{A}_{ii}})^{-1}\mathbf{A}_{\star i}$ inherits properties from $\mathbf{A}$

✓ **Nonnegativity:** $\mathbf{A} \geq 0 \implies \mathbf{P}_{ii} \geq 0$

✓ **Irreducibility:** $\mathbf{A}$ irreducible $\implies$ $\mathbf{P}_{ii}$ irreducible

✓ **Spec Radius:** ρ = Sp Radius($\mathbf{A}$) $\implies$ ρ = Sp Radius($\mathbf{P}_{ii}$)

Uncoupling — Coupling

Perron Complement Vectors

- $\mathbf{z}_1 \quad \mathbf{z}_2 \quad \cdots \quad \mathbf{z}_k$ where $\mathbf{P}_{ii}\mathbf{z}_i = \rho\mathbf{z}_i \qquad \mathbf{z}_i > 0 \qquad \mathbf{e}^T\mathbf{z}_i = 1$

$$\mathbf{e}^T = [\,1 \quad 1 \quad \cdots \quad 1\,]$$

The Problem

- Couple $\mathbf{z}_i$'s together to build Perron vector for $\mathbf{A}$

$$\mathbf{x} = \begin{bmatrix} y_1\mathbf{z}_1 \\ y_2\mathbf{z}_2 \\ \vdots \\ y_k\mathbf{z}_k \end{bmatrix}$$

A Common Theme

Restriction Operator

$$\mathcal{R} = \begin{bmatrix} \mathbf{e}^T & & & \\ & \mathbf{e}^T & & \\ & & \ddots & \\ & & & \mathbf{e}^T \end{bmatrix}_{k \times n}$$

Prolongation Operator

$$\mathcal{P} = \begin{bmatrix} \mathbf{z}_1 & & & \\ & \mathbf{z}_2 & & \\ & & \ddots & \\ & & & \mathbf{z}_k \end{bmatrix}_{n \times k}$$

♦ $\mathcal{R}\mathcal{P} = \mathbf{I}$ ♦

Coupling Matrix

$$\mathbf{C} = \mathcal{R}\mathbf{A}\mathcal{P} = \left[\mathbf{e}^T \mathbf{A}_{ij} \mathbf{z}_j\right]_{k \times k}$$

More Inheritance

$\mathbf{C} = \mathcal{R}\mathbf{A}\mathcal{P}$ inherits properties from $\mathbf{A}$

✓ **Nonnegativity:** $\mathbf{A} \geq 0 \implies \mathbf{C} \geq 0$

✓ **Irreducibility:** $\mathbf{A}$ irreducible $\implies$ $\mathbf{C}$ irreducible

✓ **Spec Radius:** ρ = Sp Radius($\mathbf{A}$) $\implies$ ρ = Sp Radius($\mathbf{C}$)

Putting Things Together

Another P–F Problem ♦ $\mathbf{C}_{k\times k}\mathbf{y} = \rho\mathbf{y}, \quad \mathbf{y} > 0, \quad \mathbf{e}^T\mathbf{y} = 1$

Coupling Coefficients ♦ **Perron Complement Vectors**

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix} \qquad \mathbf{z} = \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \\ \vdots \\ \mathbf{z}_k \end{bmatrix}$$

Perron Vector of A

$$\mathbf{x} = \mathbf{y} \otimes \mathbf{z} = \begin{bmatrix} y_1\mathbf{z}_1 \\ y_2\mathbf{z}_2 \\ \vdots \\ y_k\mathbf{z}_k \end{bmatrix}$$

$$\textbf{For } \mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}$$

1. Form

- $\mathbf{P}_{11} = \mathbf{A}_{11} + \mathbf{A}_{12}(\rho\mathbf{I} - \mathbf{A}_{22})^{-1}\mathbf{A}_{21}$
- $\mathbf{P}_{22} = \mathbf{A}_{22} + \mathbf{A}_{21}(\rho\mathbf{I} - \mathbf{A}_{11})^{-1}\mathbf{A}_{12}$

2. Solve $\mathbf{P}_{11}\mathbf{z}_1 = \rho\mathbf{z}_1$ and $\mathbf{P}_{22}\mathbf{z}_2 = \rho\mathbf{z}_2$

3. Form $\mathbf{C}_{2\times 2} = \left[\mathbf{e}^T\mathbf{A}_{ij}\mathbf{z}_j\right]$

4. Solve $\mathbf{C}\mathbf{y} = \rho\mathbf{y}$

5. Form $\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} = \begin{bmatrix} y_1\mathbf{z}_1 \\ y_2\mathbf{z}_2 \end{bmatrix}$

1. Form
- $\mathbf{P}_{11} = \mathbf{A}_{11} + \mathbf{A}_{12}(\rho\mathbf{I} - \mathbf{A}_{22})^{-1}\mathbf{A}_{21}$
- $\mathbf{P}_{22} = \mathbf{A}_{22} + \mathbf{A}_{21}(\rho\mathbf{I} - \mathbf{A}_{11})^{-1}\mathbf{A}_{12}$

2. Solve $\mathbf{P}_{ii}\mathbf{z}_i = \rho\mathbf{z}_i$ $\longleftarrow$ Divide & Conquer
- Implement "Fork–Join"
- Makes a Parallel Algorithm

3. Form $\mathbf{C}_{2\times 2} = \left[\mathbf{e}^T\mathbf{A}_{ij}\mathbf{z}_j\right]$

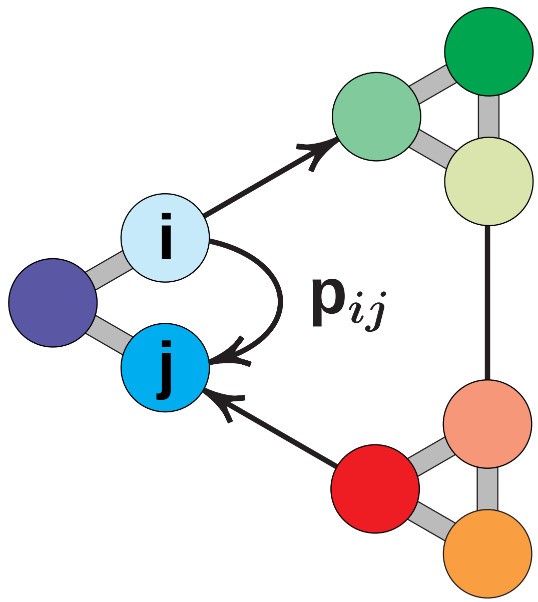
4. Solve $\mathbf{C}\mathbf{y} = \rho\mathbf{y}$ $\longleftarrow$

$$\begin{cases} y_1 = \dfrac{\mathbf{e}^T\mathbf{A}_{12}\mathbf{z}_2}{\rho - \mathbf{e}^T\mathbf{A}_{11}\mathbf{z}_1 + \mathbf{e}^T\mathbf{A}_{12}\mathbf{z}_2} \\ y_2 = 1 - y_1 \end{cases}$$

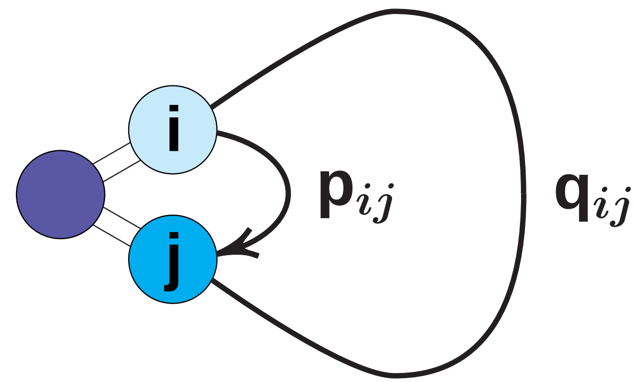
5. Form $\mathbf{x} = \begin{bmatrix}\mathbf{x}_1 \\ \mathbf{x}_2\end{bmatrix} = \begin{bmatrix} y_1\mathbf{z}_1 \\ y_2\mathbf{z}_2\end{bmatrix}$

Censoring Markov Chains

- Observe process only on a Subset (cluster) of states
- When cluster is left, go to sleep until cluster is re-entered



(UNCENSORED)



(CENSORED)

$\mathbf{p}_{ij}$ = P(i to j directly) $\mathbf{q}_{ij}$ = P(re-enter at j / leave from i)

- Censored Transition Probability $\equiv \mathbf{p}_{ij} + \mathbf{q}_{ij}$

Censored Transition Matrices

THEOREM

✓ If $\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \cdots & \mathbf{A}_{1k} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \cdots & \mathbf{A}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{k1} & \mathbf{A}_{k2} & \cdots & \mathbf{A}_{kk} \end{bmatrix}$ is the transition matrix for an irreducible chain, then the Perron (stochastic) complement

$\mathbf{P}_{ii} = \mathbf{A}_{ii} + \mathbf{A}_{i\star}(\mathbf{I} - \overline{\mathbf{A}_{ii}})^{-1}\mathbf{A}_{\star i}$ is the censored transition matrix for the i^{th} cluster

Uncoupling By Censoring

Censored Distributions

✓ $\mathbf{z}_i^T$ = stationary distribution of $\mathbf{P}_{ii}$

Coupling Matrix

✓ $\mathbf{C} = \left[\mathbf{z}_i^T \mathbf{A}_{ij} \mathbf{e}\right]$

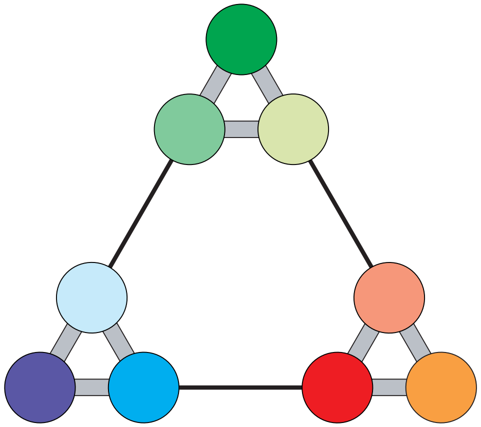
Coupling Distribution

✓ $\mathbf{y}^T = [\, y_1 \quad y_2 \quad \cdots \quad y_k \,]$ = stationary distribution of $\mathbf{C}$

Global Distribution

✓ $\mathbf{x}^T = [\, y_1 \mathbf{z}_1^T \quad y_2 \mathbf{z}_2^T \quad \cdots \quad y_k \mathbf{z}_k^T \,]$ = stationary dist of $\mathbf{A}$

Nearly Uncoupled Chains



Weakly Coupled Clusters

- $\|\mathbf{A}_{ij}\| \leq \epsilon$ for $i \neq j$

Approximate

- $\mathbf{P}_{ii} = \mathbf{A}_{ii} + \mathbf{A}_{i\star}(\mathbf{I} - \overline{\mathbf{A}_{ii}})^{-1}\mathbf{A}_{\star i} \approx \mathbf{A}_{ii} + \left(\begin{array}{c}\text{Something}\\ \text{Simple}\end{array}\right) = \widetilde{\mathbf{P}}_{ii}$

Aggregation/Disaggregation Approximations

- $\widetilde{\mathbf{P}}_{ii} \longrightarrow \widetilde{\mathbf{z}}_i^T \longrightarrow \widetilde{\mathbf{C}} \longrightarrow \widetilde{\mathbf{y}}^T \longrightarrow \widetilde{\mathbf{x}}^T = [\,\widetilde{y}_1\widetilde{\mathbf{z}}_i^T \quad \cdots \quad \widetilde{y}_k\widetilde{\mathbf{z}}_k^T\,]$

Error Analysis

- Grace Cho — NCSU