Google’s PageRank
and Beyond

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May 4, 2007
Beautiful mathematics eventually tends to be useful, and useful mathematics eventually tends to be beautiful.
Short History of IR

IR = search within doc. coll. for particular info. need (query)

B. C.  cave paintings

12th cent. A.D.  invention of paper, monks in scriptoriums

1450  Gutenberg’s printing press

1700s  Franklin’s public libraries

1872  Dewey’s decimal system

1940s-1950s  Computer
System for the Mechanical Analysis and Retrieval of Text

Harvard 1962 – 1965

Cornell 1965 – 1970

Gerard Salton

- Implemented on IBM 7094 & IBM 360
- Based on matrix methods
Term–Document Matrices

Start with dictionary of terms

Words or phrases (e.g., *landing gear*)
Term–Document Matrices

Start with dictionary of terms

Words or phrases (e.g., landing gear)

Index Each Document

Humans scour pages and mark key terms
Term–Document Matrices

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Words or phrases (e.g., landing gear)

Index Each Document
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Count $f_{ij} = \# \text{ times term } i \text{ appears in document } j$
Term–Document Matrices

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Term–Document Matrix

$$
\begin{pmatrix}
\text{Term 1} & f_{11} & f_{12} & \cdots & f_{1n} \\
\text{Term 2} & f_{21} & f_{22} & \cdots & f_{2n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\text{Term m} & f_{m1} & f_{m2} & \cdots & f_{mn}
\end{pmatrix}
= A_{m \times n}
$$
Query Matching

Query Vector

\[ q^T = (q_1, q_2, \ldots, q_m) \]

\[ q_i = \begin{cases} 
1 & \text{if Term } i \text{ is requested} \\
0 & \text{if not} 
\end{cases} \]
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How Close is Query to Each Document?
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How Close is Query to Each Document?

i.e., how close is $q$ to each column $A_i$?
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Use $$\delta_i = \cos \theta_i = \frac{q^T A_i}{\|q\| \|A_i\|}$$
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How Close is Query to Each Document?

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Use \( \delta_i = \cos \theta_i = \frac{q^T A_i}{\|q\| \|A_i\|} \)

Rank documents by size of \( \delta_i \).

Return Document \( i \) to user when \( \delta_i \geq tol \)
Susan Dumais’s Improvement

- Approximate $A$ with a lower rank matrix
- Effect is to compress data in $A$

- 2 patents for Bell/Telcordia

- LATENT SEMANTIC INDEXING
Latent Semantic Indexing

Use a Fourier expansion of $A$

$$A = \sum_{i=1}^{r} \sigma_i Z_i,$$

$$\langle Z_i | Z_j \rangle = \begin{cases} 1 & i=j, \\ 0 & i \neq j, \end{cases} \quad |\sigma_1| \geq |\sigma_2| \geq \cdots \geq |\sigma_r|$$

$$|\sigma_i| = |\langle Z_i | A \rangle| = \text{amount of } A \text{ in direction of } Z_i$$
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Realign data along dominant directions $\{Z_1, \ldots, Z_k, Z_{k+1}, \ldots, Z_r\}$

— Project $A$ onto $\text{span} \{Z_1, Z_2, \ldots, Z_k\}$
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**Truncate:**

$$A_k = P(A) = \sigma_1 Z_1 + \sigma_2 Z_2 + \cdots + \sigma_k Z_k$$
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— $Doc_2$ forced closer to $Doc_1 \implies$ better chance of finding $Doc_2$
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“Best” mathematical solution

— SVD: $A = UDV^T = \sum \sigma_i u_i v_i^T$

$Z_i = u_i v_i^T$
Strengths & Weaknesses

Pros

- Finds hidden connections
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• Finds hidden connections

• Can be adapted to identify document clusters
  — Text mining applications
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  — Can be spammed + Link structure ignored
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- Difficult to add & delete documents
- Finding optimal compression requires empirical tuning
Web Stats

Different from other document collections

- It’s huge
  - Over 10 billion pages, where average page size $\approx 500$KB
  - 20 times size of Library of Congress print collection
  - Deep Web $\approx 550$ billion pages
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- It has many users
  - Google alone processes more than 200 million queries per day
  - Approximately 0.25 sec per query involving thousands of computers
Web Search Components

Web Crawlers

Software robots gather web pages
Web Search Components

Web Crawlers
Software robots gather web pages

Doc Server Stores docs and snippits
Web Search Components

Web Crawlers
- Software robots gather web pages

Doc Server
- Stores docs and snippets

Index Server
- Scans pages and does term indexing

Terms → Pages (similar to book index)
The Ranking Module

- Measure the importance of each page
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- The measure should be Independent of any query
  - Primarily determined by the link structure of the Web
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Google’s PageRank = Google’s $$$$$$
The Process

query

Web Server
The Process

query → Web Server → Index Server
The Process

query → Web Server → Index Server → Doc Server
The Process
Business intelligence - Wikipedia, the free encyclopedia
Business intelligence (BI) is a business management term which refers to applications and technologies which are used to gather, provide access to, ... en.wikipedia.org/wiki/Business_intelligence - 43k - Cached - Similar pages

Business Intelligence .com :: The Resource for Business Intelligence
The Business Intelligence resource for business and technical professionals covering a wide range of topics including Performance Management, Data Warehouse ... www.businessintelligence.com/ - 74k - Apr 15, 2007 - Cached - Similar pages

Business Intelligence and Performance Management Software ...
Business intelligence and business performance management software. Reporting, analytics software, budgeting software, balanced scorecard software, ... www.cognos.com/ - 32k - Cached - Similar pages

Oracle Business Intelligence Solutions
The First Comprehensive, Cost-Effective BI Solution Only Oracle delivers a complete, pre-integrated technology foundation to reduce the cost and complexity ... www.oracle.com/solutions/ business_intelligence/index.html - 55k - Cached - Similar pages

Business Intelligence - Management Best Practice Reports
Business Intelligence: Providers of independent reports containing best practice advice, proprietary research findings and case studies for senior managers ... www.business-intelligence.co.uk/ - 18k - Cached - Similar pages

Intelligent Enterprise: Better Insight for Business Decisions
Take Your Pick

Amount of Internet search results that Web surfers typically scan before selecting one

- First page of search results: 39%
- First two pages: 19%
- First three pages: 9%
- More than first three pages: 10%
- A few search results*: 23%

*Top results without reading through the whole page

Note: Sample size is 2,369 people
Sources: JupiterResearch; iProspect
business intelligence - Wikipedia, the free encyclopedia
Business intelligence (BI) is a business management term which refers to applications and technologies which are used to gather, provide access to, ... en.wikipedia.org/wiki/Business_intelligence - 43k - Cached - Similar pages

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Intelligent Enterprise: Better Insight for Business Decisions
Yahoo Ad System Fails to Lift Net

Revenue Growth Declines; Project Benefits Are Seen Ramping Up in 2nd Period

By Kevin J. Delaney

Yahoo Inc. recently overhauled its online advertising system, giving some investors hope for a positive earnings surprise. So far, that hope hasn't materialized.

The Sunnyvale, Calif., company reported an 11% drop in first-quarter profit as its revenue growth rate continued a steady decline. Yahoo's shares fell about 8% in after-hours trading.

Some investors had raised hopes for the company's first-quarter results following a major overhaul of Yahoo's online advertising system dubbed Project Panama that was rolled out in recent months. But Yahoo's revenue was in line with its earlier projection, and it stuck to its outlook for the year. The company reiterated earlier predictions that financial benefits from Panama, which includes big changes to its search-ad system designed to boost Yahoo revenue, will start kicking in during the second quarter.

Analysts said the first quarter had been expected to be a tough one when compared with earlier quarters, with benefits from Panama not yet arriving and increased competition for the graphical display advertising that some estimate represents about one-third of Yahoo's revenue. In addition, the first quarter of last year included revenue from ad brokering for Microsoft Corp., which has since been discontinued, making for tougher comparisons.

When commissions paid to marketing partners were factored out, Yahoo reported revenue of $1.18 billion for the first quarter, in line with its projection of $1.12 billion to $1.23 billion. Yahoo stuck to its prediction of 2007 revenue on that basis of $4.95 billion to $5.45 billion.

Yahoo reported its results after regular trading hours. In 4 p.m. Nasdaq Stock Market composite trading, shares were up 48 cents to $32.09. That is about 25% higher than their level at the beginning of the year and 4% above 12 months earlier. In after-hours trading, Yahoo shares fell about 8% to $29.51.

“People were expecting a possibility of upward guidance and we didn't get that so the stock is giving back some of its recent gains,” said Rob Sanderson, an analyst at American Technology Research. “This should be the toughest quarter; that was the expectation going in.”

Revenue growth continued to decline at Yahoo. Revenue rose 7% in the first quarter, ads such amid companies, I am will see in the company's stock price.

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Google profit up 69 percent

Rival Yahoo's earnings fall

THE ASSOCIATED PRESS

SAN FRANCISCO — Google's first-quarter profit rose 69 percent, maintaining the online search leader's penchant for obliterating analyst estimates.

The stellar results released Thursday left little doubt that Google has widened its lead over its closest rival in Internet search and advertising, Yahoo, whose first-quarter earnings eroded.

Google detailed its sparkling performance on the same day that several major U.S. newspaper companies announced another quarter of financial decay, underscoring an advertising shift that is enriching Internet upstarts at the expense of traditional media outlets.

Born less than a decade ago, Google now reigns as the most profitable — and probably most powerful — force on the Web. In the latest demonstration of its clout, Google earned $1 billion, or $3.18 cents per share, during the first three months of the year. That compared with net income of $523.3 million, or $1.05 per share, in the same period last year. It was also the second consecutive quarter in which Google earned $1 billion — nearly as much as the nation's largest newspaper publisher, Gannett, made all of last year.

If not for expenses incurred for employee stock compensation, Google would have earned $3.68 per share.

Quarterly revenue reached a new company high of $3.66 billion, a 53 percent increase. After subtracting advertising commissions and other payments to its partners, Google's revenue totaled $2.53 billion.

Pleasant earnings surprises have become routine for Google, which has succeeded in beating analyst estimates in all but one of 11 quarters since its ballyhooed initial public offering of stock in August 2004.

That track record had helped elevate Google's market value to nearly $150 billion, even before the stock price surged $12.55 in Thursday's extended trading. As usual, Google's financial power flowed from its search engine. That ubiquitous tool has become the hub of the Web's largest marketing network and appears to be getting even better at identifying the right ads to show with its search results, helping elicit more revenue-generating clicks. The paid clicks on the ads

within Google's vast network increased 52 percent for the first quarter, compared with year-ago levels. And more of the clicks are occurring on Google's own Web sites, increasing the company's profits because the revenue doesn't have to be shared with an advertising partner.

Although Google has been trying to develop revenue beyond the Internet, online advertising continues to produce virtually all of its profit. The company is expected to become even more dominant in that business with last year's $1.76 billion acquisition of online video leader YouTube and its recently announced $3.1 billion deal to buy Internet ad distributor DoubleClick.

Although Google still isn't making money from YouTube, the site is "going gangbusters," co-founder Larry Page said during Thursday's conference call.

Besides buying other companies, Google is investing heavily to accommodate its growth by hiring workers and adding computer capacity at its data centers. The company spent $597 million on capital expenditures in the first quarter and hired 1,504 employees to expand to 12,238 people.

Even so, the company ended the quarter with $11.9 billion in cash.
How To Measure “Importance”

Landmark Result Paper

Survey Paper—Big Bib
How To Measure “Importance”

Landmark Result Paper

Survey Paper—Big Bib

Authorities

Hubs
How To Measure “Importance”

- Good hubs point to good authorities
- Good authorities are pointed to by good hubs
HITS

Hypertext Induced Topic Search (1998)

Determine Authority & Hub Scores

• $a_i = \text{authority score for } P_i$
• $h_i = \text{hub score for } P_i$
Determine Authority & Hub Scores

- $a_i = \text{authority score for } P_i$
- $h_i = \text{hub score for } P_i$

Successive Refinement

- Start with $h_i = 1$ for all pages $P_i$  

\[
\mathbf{h}_0 = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}
\]
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- Start with $h_i = 1$ for all pages $P_i$  ⇒  $h_0 = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$
- Define Authority Scores (on the first pass)

$$a_i = \sum_{j: P_j \rightarrow P_i} h_j$$
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Determine Authority & Hub Scores
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\[
\begin{align*}
 a_i &= \sum_{j: P_j \rightarrow P_i} h_j \\
 \Rightarrow \quad a_1 &= \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = L^T h_0 \\
 L_{ij} &= \begin{cases} 
 1 & P_i \rightarrow P_j \\
 0 & P_i \not\rightarrow P_j 
\end{cases}
\end{align*}
\]
Refine Hub Scores

- \( h_i = \sum_{j: P_i \rightarrow P_j} a_j \) \( \Rightarrow \) \( h_1 = La_1 \)

\[
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HITS Algorithm

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Successively Re-refine Authority & Hub Scores

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HITS Algorithm

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  - \( a_2 = L^T h_1 \)
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  \ldots
HITS Algorithm

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• \( a_1 = L^T h_0 \)
  • \( h_1 = L a_1 \)
    • \( a_2 = L^T h_1 \)
      • \( h_2 = L a_2 \)

Combined Iterations

• \( A = L^T L \) (authority matrix)
HITS Algorithm

Refine Hub Scores

\[ h_i = \sum_{j: P_i \rightarrow P_j} a_j \quad \Rightarrow \quad h_1 = La_1 \]

\[ L_{ij} = \begin{cases} 1 & P_i \rightarrow P_j \\ 0 & P_i \not\rightarrow P_j \end{cases} \]

Successively Re-refine Authority & Hub Scores

\[ a_1 = L^T h_0 \]

\[ h_1 = La_1 \]

\[ a_2 = L^T h_1 \]

\[ h_2 = La_2 \]

Combined Iterations

\[ A = L^T L \text{ (authority matrix)} \quad a_k = A a_{k-1} \rightarrow \text{e-vector} \quad \text{ (direction)} \]
HITS Algorithm

Refine Hub Scores

- \( h_i = \sum_{j:P_i \rightarrow P_j} a_j \Rightarrow h_1 = La_1 \)

Successively Re-refine Authority & Hub Scores

- \( a_1 = L^T h_0 \)
  - \( h_1 = La_1 \)
    - \( a_2 = L^T h_1 \)
      - \( h_2 = La_2 \)

Combined Iterations

- \( A = L^T L \) (authority matrix) \( a_k = Aa_{k-1} \rightarrow \) e-vector (direction)
- \( H = LL^T \) (hub matrix) \( h_k = Hh_{k-1} \rightarrow \) e-vector (direction)

\[ L_{ij} = \begin{cases} 1 & P_i \rightarrow P_j \\ 0 & P_i \not\rightarrow P_j \end{cases} \]
HITS Algorithm

Refine Hub Scores

\[ h_i = \sum_{j: P_i \to P_j} a_j \quad \Rightarrow \quad h_1 = La_1 \]

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Successively Re-refine Authority & Hub Scores

\[ a_1 = L^T h_0 \]

\[ h_1 = La_1 \]

\[ a_2 = L^T h_1 \]

\[ h_2 = La_2 \]

Combined Iterations

\[ A = L^T L \quad \text{(authority matrix)} \quad a_k = Aa_{k-1} \quad \rightarrow \quad \text{e-vector (direction)} \]

\[ H = LL^T \quad \text{(hub matrix)} \quad h_k = Hh_{k-1} \quad \rightarrow \quad \text{e-vector (direction)} \]

!! May not be uniquely defined if A or H is reducible !!
Compromise

1. Do direct query matching
Compromise

1. Do direct query matching

2. Build neighborhood graph
Compromise

1. Do direct query matching

2. Build neighborhood graph

3. Compute authority & hub scores for just the neighborhood
Pros & Cons

Advantages

• Returns satisfactory results
  — Client gets both authority & hub scores
Pros & Cons

Advantages

• Returns satisfactory results
  — Client gets both authority & hub scores

• Some flexibility for making refinements
Pros & Cons

Advantages

- Returns satisfactory results
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Disadvantages

- Too much has to happen while client is waiting
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  - Custom built neighborhood graph needed for each query
Pros & Cons

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  — Two eigenvector computations needed for each query
Pros & Cons

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Disadvantages

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  - Custom built neighborhood graph needed for each query
  - Two eigenvector computations needed for each query
- Scores can be manipulated by creating artificial hubs
HITS Applied
The Next Frontiers

The New Age of Google

The Search Giant Has Changed Our Lives. Can Anybody Catch These Guys? By Steven Levy

PLUS: The Future of Digital Voting

Google founders Larry Page and Sergey Brin
Google’s PageRank

(Lawrence Page & Sergey Brin 1998)

The Google Goals

- Create a PageRank \( r(P) \) that is not query dependent
  - Off-line calculations — No query time computation

- Let the Web vote with in-links
  - But not by simple link counts
    - One link to \( P \) from Yahoo! is important
    - Many links to \( P \) from me is not

- Share The Vote
  - Yahoo! casts many “votes”
    - value of vote from \( Yahoo! \) is diluted
  - If Yahoo! “votes” for \( n \) pages
    - Then \( P \) receives only \( r(Y)/n \) credit from \( Y \)
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PageRank

The Definition

\[ r(P) = \sum_{P \in B_P} \frac{r(P)}{|P|} \]

\[ B_P = \{ \text{all pages pointing to } P \} \]

\[ |P| = \text{number of out links from } P \]
PageRank

The Definition

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Successive Refinement

Start with \( r_0(P_i) = 1/n \) for all pages \( P_1, P_2, \ldots, P_n \)
PageRank

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Start with \( r_0(P_i) = \frac{1}{n} \) for all pages \( P_1, P_2, \ldots, P_n \)

Iteratively refine rankings for each page

\[ r_1(P_i) = \sum_{P \in B_{P_i}} \frac{r_0(P)}{|P|} \]
PageRank

The Definition

\[ r(P) = \sum_{P \in B_P} \frac{r(P)}{|P|} \]

\( B_P = \{\text{all pages pointing to } P\} \)

\( |P| = \text{number of out links from } P \)

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\[ r_1(P_i) = \sum_{P \in B_{P_i}} \frac{r_0(P)}{|P|} \]

\[ r_2(P_i) = \sum_{P \in B_{P_i}} \frac{r_1(P)}{|P|} \]
PageRank

The Definition

\[ r(P) = \sum_{P \in \mathcal{B}_P} \frac{r(P)}{|P|} \]

\( \mathcal{B}_P = \{ \text{all pages pointing to } P \} \)

\(|P| = \text{number of out links from } P \)

Successive Refinement

Start with \( r_0(P_i) = \frac{1}{n} \) for all pages \( P_1, P_2, \ldots, P_n \)

Iteratively refine rankings for each page

\[ r_1(P_i) = \sum_{P \in \mathcal{B}_{P_i}} \frac{r_0(P)}{|P|} \]

\[ r_2(P_i) = \sum_{P \in \mathcal{B}_{P_i}} \frac{r_1(P)}{|P|} \]

\[ \vdots \]

\[ r_{j+1}(P_i) = \sum_{P \in \mathcal{B}_{P_i}} \frac{r_j(P)}{|P|} \]
In Matrix Notation

After Step \( k \)

\[ \pi_k^T = [r_k(P_1), r_k(P_2), \ldots, r_k(P_n)] \]
In Matrix Notation

After Step $k$

$\pi^T_k = [r_k(P_1), r_k(P_2), \cdots, r_k(P_n)]$

$\pi^T_{k+1} = \pi^T_k H$ where

$h_{ij} = \begin{cases} 
\frac{1}{|P_i|} & \text{if } i \rightarrow j \\
0 & \text{otherwise}
\end{cases}$
In Matrix Notation

After Step $k$

$\pi_k^T = [r_k(P_1), r_k(P_2), \ldots, r_k(P_n)]$

$\pi_{k+1}^T = \pi_k^T H$  where $h_{ij} = \begin{cases} 
1/|P_i| & \text{if } i \to j \\
0 & \text{otherwise}
\end{cases}$

PageRank vector $= \pi^T = \lim_{k \to \infty} \pi_k^T$ = eigenvector for $H$

Provided that the limit exists
Tiny Web

\[
\mathbf{H} = \begin{pmatrix}
P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\
P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\
P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\
P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\
P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\
P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\
\end{pmatrix}
\]
Tiny Web

\[
\begin{pmatrix}
P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\
0 & 1/2 & 1/2 & 0 & 0 & 0
\end{pmatrix}
\]
Tiny Web

\[ H = \begin{pmatrix} P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\ P_1 & 0 & 1/2 & 1/2 & 0 & 0 & 0 \\ P_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ P_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ P_4 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \]
Tiny Web

\[
H = \begin{pmatrix}
P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\
0 & 1/2 & 1/2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1/3 & 1/3 & 0 & 0 & 1/3 & 0 \\
\end{pmatrix}
\]
Tiny Web

\[ H = \begin{pmatrix}
P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\
0 & 1/2 & 1/2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1/3 & 1/3 & 0 & 0 & 1/3 & 0 \\
0 & 0 & 0 & 0 & 1/2 & 1/2
\end{pmatrix} \]
## Tiny Web

The adjacency matrix $H$ for the Tiny Web is given by:

$$
H = \begin{pmatrix}
P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\
0 & 1/2 & 1/2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1/3 & 1/3 & 0 & 0 & 1/3 & 0 \\
0 & 0 & 0 & 0 & 1/2 & 1/2 \\
0 & 0 & 0 & 1/2 & 0 & 1/2
\end{pmatrix}
$$

The diagram shows the web structure with arrows indicating the direction of links between nodes 1, 2, 3, 4, 5, and 6.
\[ H = \begin{pmatrix} P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\ P_1 & 0 & 1/2 & 1/2 & 0 & 0 & 0 \\ P_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ P_3 & 1/3 & 1/3 & 0 & 0 & 1/3 & 0 \\ P_4 & 0 & 0 & 0 & 0 & 1/2 & 1/2 \\ P_5 & 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ P_6 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \]
Tiny Web

\[ H = \begin{pmatrix} P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\ P_1 & 0 & 1/2 & 1/2 & 0 & 0 & 0 \\ P_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ P_3 & 1/3 & 1/3 & 0 & 0 & 1/3 & 0 \\ P_4 & 0 & 0 & 0 & 0 & 1/2 & 1/2 \\ P_5 & 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ P_6 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \]

▷ A random walk on the Web Graph
A random walk on the Web Graph

PageRank = \( \pi_i \) = amount of time spent at \( P_i \)
A random walk on the Web Graph

PageRank = $\pi_i$ = amount of time spent at $P_i$

Dead end page (nothing to click on) — a “dangling node”
Tiny Web

\[ H = \begin{pmatrix}
P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\
\begin{array}{cccccc}
P_1 & 0 & 1/2 & 1/2 & 0 & 0 & 0 \\
P_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
P_3 & 1/3 & 1/3 & 0 & 0 & 1/3 & 0 \\
P_4 & 0 & 0 & 0 & 0 & 1/2 & 1/2 \\
P_5 & 0 & 0 & 0 & 1/2 & 0 & 1/2 \\
P_6 & 0 & 0 & 0 & 1 & 0 & 0 \\
\end{array}
\end{pmatrix} \]

- A random walk on the Web Graph
- PageRank = \( \pi_i \) = amount of time spent at \( P_i \)
- Dead end page (nothing to click on) — a “dangling node”
- \( \pi^T = (0, 1, 0, 0, 0, 0) = e\text{-vector} \Rightarrow \text{Page } P_2 \text{ is a “rank sink”} \)
The Fix

Allow Web Surfers To Make Random Jumps
The Fix

Allow Web Surfers To Make Random Jumps

Replace zero rows with \( \frac{e^T}{n} = \left( \frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n} \right) \)

\[
\mathbf{s} = \begin{pmatrix}
P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\
0 & 1/2 & 1/2 & 0 & 0 & 0 \\
1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\
1/3 & 1/3 & 0 & 0 & 1/3 & 0 \\
0 & 0 & 0 & 0 & 1/2 & 1/2 \\
0 & 0 & 0 & 1/2 & 0 & 1/2 \\
0 & 0 & 0 & 1 & 0 & 0
\end{pmatrix}
\]
The Fix

Allow Web Surfers To Make Random Jumps

— Replace zero rows with \( \mathbf{e}^T = \left( \frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n} \right) \)

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0 & 1/2 & 1/2 & 0 & 0 & 0 \\
1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\
1/3 & 1/3 & 0 & 0 & 1/3 & 0 \\
0 & 0 & 0 & 0 & 1/2 & 1/2 \\
0 & 0 & 0 & 1/2 & 0 & 1/2 \\
0 & 0 & 0 & 1 & 0 & 0 \\
\end{pmatrix}
\]

— \( \mathbf{S} = \mathbf{H} + \frac{\mathbf{a} \mathbf{e}^T}{6} \) is now row stochastic \( \Rightarrow \rho(\mathbf{S}) = 1 \)
The Fix

Allow Web Surfers To Make Random Jumps

— Replace zero rows with \( \frac{e^T}{n} = \left( \frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n} \right) \)

\[
S = \begin{bmatrix}
P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\
0 & 1/2 & 1/2 & 0 & 0 & 0 \\
1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\
1/3 & 1/3 & 0 & 0 & 1/3 & 0 \\
0 & 0 & 0 & 0 & 1/2 & 1/2 \\
0 & 0 & 0 & 1/2 & 0 & 1/2 \\
0 & 0 & 0 & 1 & 0 & 0 \\
\end{bmatrix}
\]

— \( S = H + \frac{a e^T}{6} \) is now row stochastic \( \implies \rho(S) = 1 \)

— Perron says \( \exists \ \pi^T \geq 0 \ \text{s.t.} \ \pi^T = \pi^T S \ \text{with} \ \sum_i \pi_i = 1 \)
Nasty Problem

The Web Is Not Strongly Connected
Nasty Problem

The Web Is Not Strongly Connected

S is reducible

\[
S = \begin{pmatrix}
P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\
0 & 1/2 & 1/2 & 0 & 0 & 0 \\
1/6 & 1/6 & 1/6 & 1/6 & 0 & 0 \\
1/3 & 1/3 & 0 & 0 & 1/3 & 0 \\
0 & 0 & 0 & 0 & 1/2 & 1/2 \\
0 & 0 & 0 & 1/2 & 0 & 1/2 \\
0 & 0 & 0 & 1 & 0 & 0 
\end{pmatrix}
\]
Nasty Problem

The Web Is Not Strongly Connected

- $S$ is reducible

$$
S = \begin{pmatrix}
P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\
P_1 & 0 & 1/2 & 1/2 & 0 & 0 & 0 \\
P_2 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\
P_3 & 1/3 & 1/3 & 0 & 0 & 1/3 & 0 \\
P_4 & 0 & 0 & 0 & 0 & 1/2 & 1/2 \\
P_5 & 0 & 0 & 0 & 1/2 & 0 & 1/2 \\
P_6 & 0 & 0 & 0 & 1 & 0 & 0
\end{pmatrix}
$$

- Reducible $\implies$ PageRank vector is not well defined

- Frobenius says $S$ needs to be \textit{irreducible} to ensure a unique $\pi^T > 0$ s.t. $\pi^T = \pi^T S$ with $\sum_i \pi_i = 1$
Irreducibility Is Not Enough

Could Get Trapped Into A Cycle \((P_i \rightarrow P_j \rightarrow P_i)\)
Irreducibility Is Not Enough

Could Get Trapped Into A Cycle \((P_i \rightarrow P_j \rightarrow P_i)\)

The powers \(S^k\) fail to converge
Irreducibility Is Not Enough

Could Get Trapped Into A Cycle \((P_i \rightarrow P_j \rightarrow P_i)\)

--- The powers \(S^k\) fail to converge

--- \(\pi_{k+1}^T = \pi_k^T S\) fails to convergence
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Could Get Trapped Into A Cycle \((P_i \rightarrow P_j \rightarrow P_i)\)

- The powers \(S^k\) fail to converge
- \(\pi_{k+1}^T = \pi_k^T S\) fails to converge

Convergence Requirement

- Perron–Frobenius requires \(S\) to be primitive
Irreducibility Is Not Enough

Could Get Trapped Into A Cycle \((P_i \rightarrow P_j \rightarrow P_i)\)

- The powers \(S^k\) fail to converge
- \(\pi^{T}_{k+1} = \pi^{T}_k S\) fails to convergence

**Convergence Requirement**

- Perron–Frobenius requires \(S\) to be primitive
  - No eigenvalues other than \(\lambda = 1\) on unit circle
Irreducibility Is Not Enough

Could Get Trapped Into A Cycle \((P_i \rightarrow P_j \rightarrow P_i)\)

— The powers \(S^k\) fail to converge

— \(\pi^T_{k+1} = \pi^T_k S\) fails to convergence

Convergence Requirement

— Perron–Frobenius requires \(S\) to be primitive

— No eigenvalues other than \(\lambda = 1\) on unit circle

— Frobenius proved \(S\) is primitive \(\iff S^k > 0\) for some \(k\)
The Google Fix

Allow A Random Jump From Any Page

\[ G = \alpha S + (1 - \alpha)E > 0, \quad E = ee^T/n, \quad 0 < \alpha < 1 \]
The Google Fix

Allow A Random Jump From Any Page

\[ G = \alpha S + (1 - \alpha)E > 0, \quad E = ee^T/n, \quad 0 < \alpha < 1 \]

\[ G = \alpha H + uv^T > 0 \quad u = \alpha a + (1 - \alpha)e, \quad v^T = e^T/n \]
The Google Fix

Allow A Random Jump From Any Page

\[ \begin{align*}
G &= \alpha S + (1 - \alpha)E > 0, \quad E = ee^T/n, \quad 0 < \alpha < 1 \\
G &= \alpha H + uv^T > 0 \quad u = \alpha a + (1 - \alpha)e, \quad v^T = e^T/n \\
\text{PageRank vector} \quad \pi^T = \text{left-hand Perron vector of } G
\end{align*} \]
The Google Fix

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PageRank vector \[ \pi^T = \text{left-hand Perron vector of } G \]

Some Happy Accidents

\[ x^T G = \alpha x^T H + \beta v^T \]

Sparse computations with the original link structure
The Google Fix

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PageRank vector \( \pi^T \) = left-hand Perron vector of \( G \)

Some Happy Accidents

\[ x^T G = \alpha x^T H + \beta v^T \] Sparse computations with the original link structure

\[ \lambda_2(G) = \alpha \] Convergence rate controllable by Google engineers
The Google Fix

Allow A Random Jump From Any Page

— $G = \alpha S + (1 - \alpha)E > 0, \quad E = ee^T/n, \quad 0 < \alpha < 1$

— $G = \alpha H + uv^T > 0 \quad u = \alpha a + (1 - \alpha)e, \quad v^T = e^T/n$

— PageRank vector $\pi^T = \text{left-hand Perron vector of } G$

Some Happy Accidents

— $x^T G = \alpha x^T H + \beta v^T \quad \text{Sparse computations with the original link structure}$

— $\lambda_2(G) = \alpha \quad \text{Convergence rate controllable by Google engineers}$

— $v^T$ can be any positive probability vector in $G = \alpha H + uv^T$
The Google Fix

Allow A Random Jump From Any Page

\[ G = \alpha S + (1 - \alpha)E > 0, \quad E = ee^T/n, \quad 0 < \alpha < 1 \]

\[ G = \alpha H + uv^T > 0 \]

\[ u = \alpha a + (1 - \alpha)e, \quad v^T = e^T/n \]

PageRank vector \[ \pi^T = \text{left-hand Perron vector of } G \]

Some Happy Accidents

\[ x^T G = \alpha x^T H + \beta v^T \]

Sparse computations with the original link structure

\[ \lambda_2(G) = \alpha \]

Convergence rate controllable by Google engineers

\[ v^T \text{ can be any positive probability vector in } G = \alpha H + uv^T \]

The choice of \( v^T \) allows for personalization
What’s News –

Business and Finance

BUSINESS TO BUSINESS

Bush is preparing to present Congress a huge bill for Iraq costs. The total could reach $95 billion depending on the length of the possible war and occupation. As hostilities began in the U.N. to win support for a war resolution, the president again made clear he intends to act with or without the world body’s imprimatur. Arms inspectors said Baghdad provided new data, including a report of a possible biological bomb. Gen. Franks assumed command of the war operations center in Qatar. Allied warplanes are aggressively taking out missile sites that could threaten the allied troop buildup. (Column 1 and Pages A4 and A6)

Turkey’s parliament debated legislation to let the U.S. deploy 52,000 to open a northern front. Kurdish soldiers lined up in a show of force as U.S. officials traveled into Iraq’s north for an opposition conference.

Bush to Seek up to $95 Billion To Cover Costs of War on Iraq

By Greg Jaffe
And John D. McKinnon

WASHINGTON—The Bush administration is preparing supplemental spending requests totaling as much as $95 billion for a war with Iraq, its aftermath and new expenses to fight terrorism, officials said.

The total could be as low as $60 billion because Pentagon budget planners don’t know how long a military conflict will last, whether U.S. allies will contribute more than token sums to the effort and what damage Saddam Hussein might do to his own country to retaliate against conquering forces.

Budget planners also are awaiting the outcome of an intense internal debate over whether to include $13 billion in the requests to Congress that the Pentagon says it needs to fund the broader war on terrorism, as well as for stepped-up homeland security. The White House Office of Management and Budget argues that the money might not be necessary. President Bush, Defense Secretary Donald Rumsfeld and budget director Mitchell Daniels Jr. met yesterday to discuss the matter but didn’t reach a final agreement. Mr. Rumsfeld plans to reexamine his

Web Master

As the Web spreads...

Total Internet users, by household, in millions

<table>
<thead>
<tr>
<th>Year</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997</td>
<td>12.2</td>
</tr>
<tr>
<td>1998</td>
<td>32.4</td>
</tr>
<tr>
<td>1999</td>
<td>50.1</td>
</tr>
<tr>
<td>2000</td>
<td>63.3</td>
</tr>
<tr>
<td>2001</td>
<td>73.1</td>
</tr>
<tr>
<td>2002</td>
<td>78.4</td>
</tr>
</tbody>
</table>

Google’s U.S. presence expands

Top search engines, in millions of unique visits

<table>
<thead>
<tr>
<th>Engine</th>
<th>2003</th>
</tr>
</thead>
<tbody>
<tr>
<td>Google</td>
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</tr>
<tr>
<td>Yahoo</td>
<td>36.8</td>
</tr>
<tr>
<td>MSN Search</td>
<td>35.6</td>
</tr>
<tr>
<td>AOL Search</td>
<td>22.0</td>
</tr>
<tr>
<td>Ask Jeeves</td>
<td>13.3</td>
</tr>
<tr>
<td>Overture</td>
<td>6.4</td>
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</tbody>
</table>

Top shopping-referral sites, in millions of referrals

<table>
<thead>
<tr>
<th>Engine</th>
<th>2003</th>
</tr>
</thead>
<tbody>
<tr>
<td>Google</td>
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<tr>
<td>Dealline</td>
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<tr>
<td>BizRate</td>
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</tr>
<tr>
<td>Overture</td>
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</tr>
<tr>
<td>Epinions</td>
<td>0.78</td>
</tr>
<tr>
<td>CNET</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Sources: Forrester Research; Nielsen NetRatings

Exotic Leatherwear Gets Cut Off

By Michael Totty
And Mylene Mangalindan

Joy Holman sells provocative leather clothing on the Web. She wants what nearly everyone doing business online wants: more exposure on Google. So from the time she launched exoticleatherwear.com last May, she tried all sorts of tricks to get her site to show up among the first listings when a user of Google Inc.’s popular search engine typed in “women’s leatherwear” or “leather apparel.” She buried hidden words in her Web pages intended to fool Google’s computers. She signed up with a service that promised to have hundreds of sites link to her online store—thereby boosting a crucial measure in Google’s system of ranking sites. The techniques

Cat and Mouse

As Google Becomes Web’s Gatekeeper, Sites Fight to Get In

Search Engine Punishes Firms That Try to Game System; Outlawing the ‘Link Farms’
Web Sites Fight for Prime Real Estate on Google

Continued From First Page

creating Web sites that were nothing more than collections of links to the clients’ site, called “link farms.” Since Google ranks a site largely by how many links or “votes” it gets, the link farms could boost a site’s popularity.

In a similar technique, called a link exchange, a group of unrelated sites would agree to all link to each other, thereby fooling Google into thinking the sites have a multitude of votes. Many times, they also found they could buy links to themselves to boost their rankings.

Ms. Holman, the leatherwear retailer, discovered the consequences of trying to fool Google. The 42-year-old hospital laboratory technician, who learned computer skills by troubleshooting her hospital’s equipment, operates her online apparel store as a side business that she hopes can someday replace her day job.

When she launched her Exotic Leather Wear store from her home in Mesa, Ariz., she quickly learned the importance of appearing near the top of search-engine results, especially on Google. She boned up on search techniques, visiting online discussion groups dedicated to search engines and reading what material she could find on the Web.

At first, Ms. Holman limited herself to modest changes, such as loading her page with hidden metatags that would help steer a search toward her site when a user entered words such as “halter tops” or “leather men’s clothing.” Since Google gave greater weight to metatags in determining its rankings, the efforts had little effect on her search results.

She then received an e-mail advertisement from AutomatedLinks.com, a Wirral, England, company that promised to send traffic “through the roof” by linking to more than 2,000 Web sites to hers. Aside from attracting customers, the links were designed to improve her search-engine rankings.

In theory, when Google encounters the AutomatedLinks code, it treats it as a legitimate referral to the other sites and counts them in boosting the sites’ popularity. Ms. Holman signed up with AutomatedLinks in July, she read on an online discussion group that Google objected to such link arrangements. She says she immediately striped the code from her Web pages. For a while, her site gradually worked its way up on Google search results, and business steadily improved because links to her site still remained on the sites of other AutomatedLinks customers. Then, sometime in November, her site was suddenly no longer appearing in the top results. Her orders plunged as much as 80%.

Ms. Holman, who e-mailed Google and AutomatedLinks, says she has been unable to get answers. But in the last few months, other AutomatedLinks customers say they haven’t seen their sites apparently penalized by Google. Graham McLeay, who runs a small chauffeur service north of London, saw revenue cut in half during the two months he believes his site was penalized by Google.

The high-stakes fight between Google and the optimizers can leave some Web-site owners confused. “I don’t know how people are supposed to judge what is right and wrong,” says Mr. McLeay.

AutomatedLinks didn’t respond to requests for comment. Google, however, says Mr. McLeay, the Google engineer, warns that the rules are clear and that it’s better to follow them rather than try to get a problem fixed after a site has been penalized. “We want to return the most relevant pages we can,” Mr. McLeay says. “The best way for a site owner to do that is follow our guidelines.”

Crackdown

Google has been stepping up its enforcement since 2001. It warned Webmasters that could get their sites kicked out of the Google index and it provided a list of forbidden activities, including hiding text and “link schemes,” such as the link farms. Google also warn against “cloaking”—showing one site’s design to score well while giving visitors a different, more attractive page—or creating multiple Web addresses that take visitors to a single site.

According to one study, about a third of the Web-home City-based SearchKing, an online directory for hundreds of small, specialty Web sites. SearchKing also sells advertising links designed both to raise an advertiser’s rank in Google and other search results.

Bob Massa, SearchKing’s chief executive, last August launched the PR Ad Network as a way to capitalize on Google’s page-ranking system, which is used to evaluate PageRank, the search engine’s measure of a site’s importance, by placing ads at the top of their search results. And, the rankings can be viewed by Web users if they install special Google software. PR Ad Network sells ads that are priced according to a site’s PageRank, with higher-ranked sites commanding higher prices. When a site buys an advertising link on a highly ranked site, the ad buyer could see its ratings improve because of the greater weight Google gives to that link.

Shortly after publicizing the ad network, Mr. Massa discovered that his site suddenly dropped in Google’s rankings. What’s more, sites that participated in the separate SearchKing directory also had their Google rankings lowered. He filed a lawsuit in Oklahoma City federal court, claiming Google was punishing him for trying to profit from the company’s page-ranking system. A Google spokesman won’t comment on the case. In its court filings, Google said it deleted pages from the SearchKing site because of SearchKing’s attempts to manipulate search results. The company has asked for the suit to be dismissed, arguing that the PageRank represents its opinion of the value of a Web site and as such is protected by the First Amendment.

“arbitrage the big search engines determine the laws of how commerce runs,” says Mr. Massa.

By CHAD THREH

Home Depot Amid First

ATLANTA—Home Depot’s fiscal fourth-quarter earnings fell 3.4% on disappointing sales. Speaking to investors at an analyst, the company’s chief executive, Bob Nardelli, Home Depot is preparing to face a stream of disaffected customers and competitive challenges from rival retailers, including stores like the Home Depot is expected to face a seasonally higher level of sales, helped to improve its first-quarter results. The retailer said net income for the first quarter was up 2.9% to $3.6 billion from $3.5 billion a year earlier. The company said its operating income increased 5% and net income rose to $2.7 billion, or $1.12 a share, from $2.5 billion, or $1.05 a share.

*Ralph Lauten .*

Fiat Patria Is Set to Be

By ALESSANDRA GAI

ROME—Umberto Agnelli named Fiat SpA chairman on Feb. 2, giving into the driver’s seat as the company works on an 11th-b hour of its profitable car unit. Mr. Agnelli, the 85-year-old Fiat patriarcha Gianni Agnelli last month, was widely expected to take over from current chief of the company, Sergio Marchionne, who is set to retire later this year. Mr. Marchionne, who has served as chairman of Fiat since 1994, is expected to take over as CEO of Chrysler Group LLC, the U.S. arm of Italian automaker Fiat SpA, when the company completes its deal to acquire Chrysler.

Mr. Agnelli, who is 85, is the third generation of the Agnelli family to run Fiat, Italy’s biggest company. Mr. Agnelli has been involved in the company’s management since the 1960s and has been serving as a board member since 1994. He has been a key figure in the company’s expansion in the 1980s and 1990s, when Fiat SpA acquired groups such as Alfa Romeo, Lamborghini and Fiat Auto. Mr. Agnelli has also been involved in the company’s efforts to restructure and improve its performance in recent years, including the company’s move to cut costs and boost efficiency.

While Mr. Agnelli is known for his business acumen, he has also been criticized for his management style, which some have described as hands-off and laissez-faire. However, Mr. Agnelli has defended his management style, saying that he believes in giving managers the freedom to make decisions and run the company as they see fit.

Mr. Agnelli’s appointment as chairman of Fiat SpA is seen as a move to bring new energy and direction to the company. The company has been facing challenges in recent years, including a decline in sales and profits, as well as a struggle to compete with other major players in the automotive industry. Mr. Agnelli is expected to work closely with the company’s CEO, Sergio Marchionne, to help the company navigate these challenges.

Mr. Agnelli has also been involved in other areas of the family’s business interests, including in the energy sector, where he has been a board member of energy companies such as Edison SpA and Enel SpA. He has also been involved in the company’s investment in renewable energy projects.

Fiat SpA is a major player in the global automotive industry, with operations in Europe, Asia, and the Americas. The company is known for its iconic brands such as Fiat, Lancia, Alfa Romeo, and Maserati, as well as for its ownership of Chrysler Group LLC, the largest U.S. automaker.
Personalization is Coming

The Wall Street Journal

Search Engines Seek to Get Inside Your Head

Google, Others Start to Comb Users’ Online Habits to Tailor Results to Personal Interests

By JESSICA E. VASCCELLARO
And KEVIN J. DELANEY

Search engines have long generated the same results for queries whether the person searching was a mom, mathematician or movie star. Now, who you are and what you’re interested in is starting to affect the outcome of your search.

Google Inc. and a wide range of start-ups are trying to translate factors like where you live, the ads you click on and the types of restaurants you search for into more-relevant search results. A chef who searched for “beef,” for example, might be more likely to find recipes than encyclopedia entries about livestock. And a film buff who searched for a new movie might see detailed articles about the making of the film, rather than ticket-buying sites.

Google has been enhancing and more widely deploying its search-personalization technology. Within coming weeks, Google users who are logged in will begin having their search results reordered based on information they have provided to Google. For instance, they may have entered a city to receive weather forecasts on a personalized Google home page. As a result, a user in New York who types in “Giants” might see higher search results for the football team than a user in San Francisco, who might be more interested in the Giants baseball team.

Consumers who use its Web-history service to track previous search queries currently get results that are influenced by those queries and the sites they have clicked on. The company plans eventually to offer personalization based on a user’s Web-browsing history—including sites people visited without going through Google—when users agree to let Google track it.

Also, within three to five years, Google will
Conclusion

Google Augments PR With Content Scores For Final Rankings

Content “Metrics” Are Proprietary — But Known Examples

- Whether query terms appear in the title or the body
- Number of times query terms appear in a page
- Proximity of multiple query words to one another
- Appearance of query terms in a page (e.g., headings in bold font score higher)
- Content of neighboring web pages

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